

Assertion and Conceptual Roles

The paradigmatic linguistic activity is saying that-p, in the sense of asserting, claiming, or stating that-p for some declarative sentence p. Frege shows in the Begriffsschrift that the ways in which sentences can occur as significant constituents of other sentences requires us to distinguish the content of such an assertion (what is asserted) and the force of the assertion (the asserting of that content). For when a sentence appears as the antecedent of an asserted conditional, it must have something, let us call it the 'content', in common with its occurrence as a free-standing assertion, or there would be no justification for detaching the consequent of the conditional when one is prepared to assert its antecedent. On the other hand, the asserting of the conditional does not include the asserting of the antecedent, since the assertor of the conditional might well take the former to be true and latter to be false. It is a criterion of adequacy for any account of either of these features of declarative discourse that it be compatible with some correct account of the other. Exclusive attention to the practice of asserting precludes understanding the conceptual significance which such linguistic performances express and enable, while the complementary exclusion must cut off semantic theory from its only empirical subject-matter, talking as something people do.

The most familiar strategy for addressing these two questions accords explanatory priority to the notion of content which sentences of the same type generally share whether occurring as significant components of other sentences or not.¹ This approach envisages associating abstract objects of some sort with sentences, and defining an algebra on those objects which illuminates the varieties of combination of sentences. An account of the use in assertion of sentences associated with such contents is then a further project, which will draw on the resources of the prior semantic theory. Such a semantic theory may have taken the form of a presentation of all sentences as species of, say, representation, or classification (or, as for the later Frege, identification of the form 'p=the true'), and it might be tempting to think that such a theory offers special resources for a theory of asserting as representing, classifying, or identifying. It is important to realize that the same considerations which disclose the distinction of force and content in the first place expose such advantages as spurious. Whether the projected account takes the form of identify-

ing asserting that Pynchon is a great novelist with classifying (representing) him as such, classifying (representing) the sentence 'Pynchon is a great novelist' as true, or classifying the actual world as a member of the set of possible worlds determined by the proposition which is the intension of that sentence, the same dilemma will be faced. Either in asserting that if Pynchon is a great novelist, then so is Hunter S. Thompson, I am performing the appropriate classification (of Pynchon, the antecedent sentence, the actual world, etc.), in which case the account gives the wrong results about asserting, or I am not performing it, in which case it is clear that appeal is made to another force-nation, classifying, which is not illuminated by the employment of its homonym in semantic theory. In the sense in which classifying s as a P can be identified with asserting that s is a P, the capacity to say in the former idiom what we are accustomed to saying in the latter represents progress only insofar as it is easier to develop an adequate account of the activity of classifying (representing, identifying, etc.) than of the activity of asserting. There is no reason to suppose that the semantic representability of all sentences in terms of, say, set-membership statements or identity statements, reflects or is reflected in the explanatory priority of various kinds of linguistic performances.²

Grice, Lewis and Searle offer genuine accounts of assertion which are valuable especially for their detailed descriptions of various ways in which new sorts of performances are made possible by the collaboration of social convention and semantically contentful states of individuals such as beliefs and intentions.³ Yet the contrast between performances and their semantic contents which leads to the need for a theory of assertion separate from semantic theory in the first place is mirrored at the psychological level in the distinction between a state of belief or believing (intention or intending) and the content of that state (manifested in sententially compound beliefs). Whether removing the issue from the realm of social performances to that of psychological states represents analytic progress of strategic importance depends of course on what sort of account it is possible to give of such psychological states, and their possession of semantic content of the sort indicated in English by 'that'-clauses containing embedded sentences.

The semantic theory is to specify first what a content is (e.g. what kind of abstract object), and the theories of assertion under consideration explain the association of such contents with the utterances which are assertions in terms of the relation of those utterances to various psychological states which

we suppose already to be associated with their contents, together with a context of convention. To complete the theory of linguistic activity in accord with this strategy, one would need to account for the individuation and the acquisition of content by these states. One promising line of attack on this problem looks to functional role in the individual's cognitive economy for individuation, and to causal interaction with the environment bringing it about that such states can represent their environment.⁴

The explanatory strategy indicated by this sketch is plausible and well-developed. It is the purpose of this paper to suggest an alternative way of conceiving the relations between accounts of the performances which are assertions, states of the individual speaker-hearer which those performances express, in some sense, and the semantic contents possessed by such performances. I present first a general framework for discussing social practices, and develop in that framework a definition of assertion in terms of relations and distinctions between social performances which any theory of the practice of assertion will have to take account of. It then turns out that giving a rich enough description of the social practices involved in assertion allows us to exhibit semantic contents as complex formal features of performances and compound dispositions to perform according to those practices. In other words, I want to show that it is possible to turn exactly on its head the standard order of explanation canvassed above. The view which then emerges has a number of specific virtues, which I will recount as they become manifest, but its signal advantage is economy--any theory must give some account of the social practices we start from, and if we tell this part of the story carefully enough, we don't need to advance further considerations in our accounts of intentional states and semantic contents insofar as they matter for assertion.

I

In this section we set out the basic model of social practice which in the next section we will apply to the special case of assertional practices. As I shall use the term, a practice is a respect of similarity of potential performances issued by the members of a community, in virtue of which we can say that those performances are appropriate according to the practice in question. A social practice is such a respect of similarity induced by the responses to performances which other members of the community are disposed to make. The idea is that social properties are constituted by communal recognition of them. A performance is appropriate according to a particular social practice just in case the relevant community does (or, in suitable circumstances would) respond to it in a certain way. The responsive dispositions of the community just are the respects of similarity which determine the appropriateness of various performances. To specify a social practice is to specify the response which is the constitutive recognition of the appropriateness of performances with respect to that practice. Thus, to identify the social practice of greeting one another in some tribe is to identify what it is for them to treat some performances as a greeting. Any performance which the community recognizes as a greeting, by responding or being disposed to respond to it in the requisite way, is a greeting for that tribe. The essence of such social practices is to be socially perceived.

The recognitive responses which constitute social practices may be specifiable without reference to any community-member's response to them (killing the fatted calf as a mark of social favor, for instance), in which case we may call it 'objective'. But in the case of discursive practices, the constitutive responses will in general themselves be performances which are appropriate (in virtue of the responses the community is disposed to make to them) according to some other social practice. The appropriateness of any particular performance will then depend on the appropriateness of a whole set of other performances with similar dependences. Each social practice will definitionally depend upon a set of others. We can be a little more precise about this situation if we suppose that we have specified for us in advance first the community whose practices we care about, and second a field of possible performances by members of that community, which we can identify and individuate in some way prior to determining the role such performances might play in the set of social practices to be investigated. Then a social practice can be defined more rigorously as an ordered

pair, the intension of the practice, is the set of all possible performances which would be constitutive recognitions of the appropriateness of some element of the extension. We thus identify a set of performances with the repeatable response-type which generates the social practice by constituting a respect of similarity between prospective members of the extension. The extension consists of any performance which would be responded to by one of the performances in the intension. An objective set of performances may be considered a degenerate social practice, with a null intension.

A set of social practices defined on a universe of possible performances by community members will form a system if every practice-generating intension in the set is (a Boolean combination of) the extensions of some other practice(s) in the set.⁵ We may call an element of such a system ancestrally objective if it is either objective or its intension is the extension of some objective practice, or its intension is the extension of some practice whose intension is objective, etc.. Then notice that our definition of a system of social practices does not require that all the member practices be ancestrally objective. Definitional chains specifying the extension of one practice in terms of its intension, and that intension in terms of another extension, and so on, may loop back on one another. We will say that any system of social practices which does so, i.e., which does not consist exclusively of ancestrally objective practices is a holistic system, and that any of its practices which are not ancestrally objective are essentially holistic practices. Such a system of practices cannot be attributed to a community piecemeal, or in an heirarchic fashion, but only all at once.

It follows that in systems containing essentially holistic practices, the norma of conduct which are codified in such practices are not reducible to facts about objective performances. The appropriateness or inappropriateness of any particular performance with respect to such a practice cannot ultimately be expressed in terms of communal dispositions to respond with objectively characterizable sanctions and rewards. The only consequence of behaving inappropriately will be not being recognized as behaving appropriately, which may in turn affect the appropriateness of other performances, but need have no consequences outside the sphere of the social appraisal of appropriateness according to various social practices. The norms themselves are entirely constituted by the practices of socially recognizing performances as according or not according with them, and in the case of essentially holistic practices in a system, every such appraisal

may presuppose other such appraisals.⁶

Any interpretation according to this model of a complex of behavior as a system of social practices presupposes the previous identification of a community of individual performers and a field of possible performances by those community members. We'll discuss the second presupposition in more detail when we consider a special system of linguistic practices in the next section. For now we need say only that the performances must be objectively individuated and perceptually recognizable by members of the relevant community. Such possible performances are vehicles which, by coming to play roles in an appropriate system of social practices, acquire the social significances from which specifically linguistic significances (semantic contentents, conceptual roles) can be precipitated.

The presupposition of a community requires a little more attention, since it is the responsive dispositions of community members which determine systems of social practices such as those we will investigate. A community ought to be thought of as socially synthesized by mutual recognition of its members, since a plausible sufficient condition of A's being a member of some community is that the other members of that community take him to be such. Let us take as a primitive some response-type R of which the individuals we presuppose are capable. Then we can say that A R-recognizes a certain group of individuals, namely those he is disposed to respond to with a performance of type R. Just to recognize such a group is not yet to belong to it, however. But if A is in turn R-recognized by those whom he R-recognizes, then we can say that A belongs to the group. This simple Hegelian model of the synthesis of social entities by mutual recognition of individuals has the advantage that it preserves the basic distinction between the individual's contribution to his membership in a group and the contribution of the other members. For it is up to the individual whom to recognize (he has that much "say"), but given those recognitions, his own membership in the group requires that he in turn be recognized, a matter over which the "recognitive petitioner" may have only indirect control, or none at all. I take it to be antecedently plausible that one only has the right to treat oneself as a member of certain communities, such as the community of good chess players, fluent speakers of Hungarian, or competent philosophers insofar as one is (or under suitable circumstances, would be) recognized as such by those one recognizes as such. I may set my standards of recognition purposely low so as to make it easier to be so-recognized in turn, but then it is only in the less restrictive community that

I can claim membership. The crucial point is that reflexive recognition as (social self-recognition) be an achievement requiring the symmetry of being recognized in a particular respect by those whom I recognize in that respect, and presupposing that my recognitions will be transitive.^{6.5}

We can convert these basic considerations into a definition of a recognitive-ly structured community as follows. First, we must distinguish, as we did not in the sketch above, between basic recognitions and transitive recognitions. Basic recognitions are simply dispositions to R-respond. Transitive recognitions will be dispositions to R-respond to anyone whom someone one is disposed to R-respond to is disposed to R-respond to. For our present purposes it will suffice to define transitive recognitions extensionally in terms of basic recognitions: If we represent the set of individuals whom A is disposed to R-recognize by $R(\underline{A})$, then we can represent by $R^2(\underline{A})$ the subset of $R(\underline{A})$ such that $\underline{B} \in R^2(\underline{A})$ just in case everyone whom B R-recognizes is also R-recognized by A ($\underline{C}: \underline{C} \in R(\underline{B}) \supset \underline{C} \in R(\underline{A})$). This is de-facto transitivity of recognition. With this notion in hand, we can allow that strictly one need not be recognized by everyone whom one recognized as a member of certain community in order to be a member, for what matters is rather whether one is recognized as such by those whose recognitions one recognizes. A community is then any set P of individuals which is closed under transitive recognition, in the sense that:

- i) For all A in P, $R^2(\underline{A})$ is a subset of P
- ii) For all A, B in P, $(\underline{B} \in R^2(\underline{A})) \supset (\underline{A} \in R(\underline{B}))$.

That is, everyone transitively recognized by someone in the community is in the community, and everyone in the community is recognized by everyone they transitively recognize. Although the community is constituted by the mutual recognitions of its members, no one member is omniscient or infallible about such membership (since in general not all those R-recognized by A will be members of the community), nor is it required that everyone recognize everyone else in the community. (Since although if A transitively recognizes B and C they must recognize him, they need not transitively recognize him, and so need not recognize each other).

A community defined in this way in terms of a response-type R will accordingly exhibit a recognitive structure. A system of social practices defined on such a community may reflect this political pre-structuring in various ways. The system of linguistic practices in which assertion plays a part which we present

in the next section includes a conversational accessibility relation which offers constraints on which individuals can play various interlocutory roles. That accessibility relation codifies the recognitive structure of the linguistic community presupposed by the system of social practices under consideration. So although we will elaborate on the significance of this relation, we will not revert to a discussion of its origins. It should be pointed out in this connection that although we have been talking as if the responses which are community-constitutive respects of recognition were objective kinds, once objective respects of responsive similarity have gotten some community-generation going, other communities may be defined by responses which must themselves be performances appropriate according to a system of social practices defined on some other community.

II

In the light of these definitions, the claim that assertion is a social practice becomes a substantive thesis. The general model of social practices sets criteria of adequacy for a justification of that thesis. For if assertion is a social practice, it can be specified as part of a system of such practices by fixing its intension in terms of the extension of some other practice similarly described, that is it can be specified by giving an account of the relations of intensions and extensions of the different kinds of practices composing the system. In particular, we must be able to say what it is for members of a community to recognize (respond to) some performance as an assertion. In the case of an essentially holistic practice such as assertion, this can be done only by describing the system of practices of which it is a part.

Two primary features of the social significance of performances which are assertions orient our account of this system. First, assertion is the informational use of sentences.⁷ To capture this use in the idiom of social practices, we must specify what sort of response on the part of the relevant community members is taking a linguistic utterance as put forward as informing. What one does with information is to evaluate it and to make inferences from it. Asserting that-p is, among other things, explicitly to authorize certain inferences. (For instance, to jump ahead, if one believes that if-p-then-q, the assertor licenses the inference to q in asserting that-p). Saying this much does not yet say what the constitutive recognition of this authorizing consists in (what the intension of the practice is), however. We cannot, for instance, take that recognition to be the disposition to make the inferences so authorized, for to do so is to respond to the performance as a true assertion, and even if all assertions are, in virtue of their informing role, put forward as true, still we must distinguish the recognition of them as so put forward from their recognition as true. Our account of the authorizing of inferences will draw upon the second major feature of the social role of assertion.

This second feature is noted by Searle when he says that an assertion (among other things) "...counts as an undertaking to the effect that p represents an actual state of affairs."⁸ Leaving aside the representationalist expansion of the content ascribed, we can see in the use of the term 'undertaking' the recognition of a dimension of responsibility in assertion, coordinate with the previously indicated dimension of authority. In asserting

that-p one is committing oneself in some sense to the claim that-p. What sort of responsibility is involved? The leading idea of the present account is that it is justificatory responsibility which one undertakes by an assertion. Justification and assertion will be exhibited as essentially holistic social practices belonging to the same system of practices, internally related to one another. So the recognitive response-type which is the intension of the social practice of assertion must include recognition of the assertor as responsible for justifying his assertoric performance under suitable circumstances. Again, to say this is not to say what such recognition consists in. But it does help us see what sort of practice assertion might be.

For primitive notions of authority and responsibility can be explicated in terms of the norms of appropriate conduct codified in social practices quite generally. Consider a community with a social practice generated by (having as its intension) performances of kind K. A performance is appropriate according to the practice just in case the community is disposed to respond to it with performances of kind K. Suppose that there is some kind of performance K' such that for some set P of possible performances which are not in accord with the practice in question if performed without some previous performance (by someone) of kind K', but which do accord with the practice if preceded by a performance of kind K'. Then it seems natural to say that, at least in a broad sense, performances of kind K' authorize performances in P, with respect to the social practice whose intension is K. In virtue of the authorizing performances, the responsive dispositions of the community (of the practice-constitutive kind K) are altered, so that what previously had not been appropriate becomes so. Authority in this sense consists in the social recognition of a performance as authorizing others.

The social practical structure of the undertaking of ur-responsibility is slightly more complex.

For a community to recognize a performance as an undertaking of a responsibility with respect to a certain practice is, in the simplest case, for their practice-constitutive recognitive dispositions to alter in virtue of that performance in such a way that some different set of performances by the undertaking performance (perhaps a subset of those that would otherwise be appropriate) now (are recognized as in) accord with the practice. If in virtue of A's performance a, the set P of A's possible performances which are in accord with a particular practice shrinks to a subset P', then by a A has undertaken a stricter performative responsibility with respect to

that practice than was incumbent on him before producing a. This notion of a performance which undertakes a responsibility originates, of course, in the basic sense of responsibility codified in all social practices, namely a responsibility with respect to that practice of acting in accord with it (just as the notion of authority sketched above stems from the practice-constitutive authority of the possible performances which make up the intension of a practice). The explicit relativity to particular practices makes these very weak notions indeed (as could be said as well of 'appropriate' similarly relativized).⁹

To apply these considerations to the specifically linguistic practice of assertion requires some special assumptions about the domain of possible performances on which the system of practices is to be defined. We assume that it has been settled which possible performances are to count as utterances of the same syntactic expression-type. The content-types assumed are syntactic-phonemic not semantic (our original question is not being begged.) The stipulated pre-structuring of the performances consists only of the discrimination of them as utterances of various repeatable linguistic expressions. We presume that the community (every member of the community) can responsively discriminate (recognize) all the expression-types and all the possible performance-tokens which are utterings of them. Discriminability in the present sense requires that there be a different response (perhaps a compound one) for each expression-type and possible performance token, which is the recognition of a performance which is an utterance as of that type, or of an uttering that it is the performance-token that it is. (Presumably, spatio-temporal indicators whether in the form of demonstrative pro-performances or explicitly verbal ones will be employed by the discriminative dispositions which cognitively individuate the possible performance-tokens in the domain of the system of social practices.) Since each community member must be able to recognize what sentence is being uttered, and to tell apart occasions of utterance within such types, the responses which constitute such discriminations must themselves be repeatable across individuals.

Although the foregoing considerations do not mandate it, we will further assume that these response-types codifying the cognitive discrimination of utterance-types and possible tokenings are themselves expression-types of utterances, or recognizable features of such types. This assumption entails that recognitions of expression-types and their repeatable features accord with essentially holistic social practices, but the only details of the system contain-

ing them which we will make use of concern the expression-type (as intension) discriminability of both expression-types and tokenings (as extensions). These discriminations might be expressed by sentences of the form "He (demonstrative) just uttered the sentence _____", where the blank is filled by an utterance type-identical with that whose tokening occasions the recognitive utterance. What is essential is that the relation between the intensions and the extensions of a family of social practices underwrite a relation of what we may call (extending the usual sense) anaphoric reference between various performances. The term "anaphoric" is used to indicate that this 'referential' relation is internal to a system of social practices, where one performance refers to another as one word refers to another in A: "Pynchon wrote the book" B: "But has he tried to read it?", where the pronouns anaphorically refer to the antecedent terms 'Pynchon' and 'the book'. No relation between discursive and non-discursive items is supposed. A prime use of this expressive resource of anaphoric reference to typed utterings is exhibited just below, as a feature of demands for justification.

The key to our attempt to offer sufficient conditions for assertion by specifying a class of systems of social practices is the relation of justification which a set of assertions can have to another assertion. Performances are justifying if they are or would be treated as such by community members who are conversationally accessible from the original performers. We must specify what performances constitute the recognition that some set of utterances is a justification. Both the dimension of authority and the dimension of responsibility of assertion will be explicated in terms of the recognition of justification. Each of the different types of assertion which play a role in the systems we examine, free-standing assertions, assertions which are the results of inferences authorized by other assertions, and assertions which are part of the justification which another asserting made its assertor responsible for, each of these types of assertion incurs a justificatory responsibility itself and authorizes further inferences. The relevant responsibility is to produce (what would be recognized as) an appropriate justification, if one is demanded. We suppose that there is some conventional expression-type performing this function, incorporating features establishing its anaphoric reference to the asserting for which a justification is being petitioned. The utterance of a conventional request for justification addressed to a foregoing assertion is to be always appropriate, and not itself in need of justification. The cognitive significance of the linguistic

practices we describe stems from this universal appropriateness of demands for further justification (as Sellars takes the 'rational' structure of scientific practice to consist in its being a "self-correcting enterprise which can put any claim in jeopardy, though not all at once."¹⁰).

It is crucial to this social practice idiom that the rewards and sanctions associated with fulfilling and failing to fulfill practically generated responsibilities be differences in the appropriateness of various possible performances according to other practices. The only sanction it is appropriate to recognize in the case of a failure to fulfill the responsibility to justify a previous assertion of one's own is the withdrawal of the recognition which would otherwise be accorded to the authority of the asserting to license inferences. An utterance in the conventional style of assertions (utterances which undertake justificatory responsibilities and issue inference licenses whose contents vary as the contents of the assertion vary) will constitutively be recognized as possessing that authority only so long as the conditional responsibility to justify if queried has not been shirked. (This is an idealization of actual affairs, of course, but we are attempting in terms of social practices to give sufficient conditions for the identification of assertion, not necessary conditions.) An assertion claims an authority it may not have (if it can't be suitably justified) in the sense that if the responsibility thereby undertaken were fulfilled, then the utterance would have that authority. No more for this distinction than elsewhere in the social practice story need we appeal to intentions or beliefs of performers.

The authority of a particular asserting is whatever it is taken to be by community members conversationally accessible from the assertor. Their recognition of such authority consists in their recognition of certain justifications proffered by still other interlocutors. Still taking for granted for the moment the notion of recognizing a set of assertions as a justification of another assertion, we can put the basic idea like this. B recognizes A as having authorized inferences to q by his uttering u of p just in case any justification containing q which B will recognize he will also recognize if for q is substituted a sentence-type anaphorically referring to A's uttering u (let us represent such anaphoric reference by $\emptyset(u)$). More precisely, if $j(q)$ is some set of assertings which include an asserting of the sentence q and $B(j(q);r)$ indicates that B recognizes $j(q)$ as a justification of the sentence r (that is, would re-

cognize any tokening of $j(q)$ as a justification for any tokening of r), then if $B(j(q);r)$ entails that $B(j(\emptyset(u)/q);r)$ (where this last signifies a performance of the same type as $j(q)$, save that an uttering of type $\emptyset(u)$ has been substituted for that of type q) B recognizes A as having authorized the inference to q by the assertion u . We bow to the political structure of the community, determining whose recognitions are definitive for each practice, and specify that B must be conversationally accessible from A and that it is B's recognitions of justifications of type $j(q)$ performed by members of the community from whom B is conversationally accessible. This is just because we have called "conversational accessibility" the community-constitutive relation which determines for each individual A whose possible performances will compose the intension of basic social practices as far as concerns the generation of that portion of the extension of those practices which consists of possible performances by A.

B recognizes no authorization by u (perhaps in recognition of A's inability to fulfill his justificatory responsibility) just in case there is no q such that for all C from which B is accessible, for all r , and for all $j(q)$, if B recognizes C's utterance of $j(q)$ as justifying r then B recognizes C's utterance of $j(\emptyset(u)/j(q))$ as such a justification. Notice that this condition requires that even though u is an utterance of p , B does not recognize the conventional invocation of u in $\emptyset(u)$ as justifying C's assertion of p . That is, not only must B not take A's assertion u of p as authorizing inferences, B must not take u as authorizing even the trivial inference to p . In this situation lies the significance of failure to recognize the authority of assertion. For just as inference passes the authority of assertion one way along the anaphoric chain, it also passes the justificatory responsibility incurred the other way along that chain. When for some C from whom B is conversationally accessible $B(j(\emptyset(u)/q) r)$, B (whose recognitions of these things are what matter for C) also recognizes that where in uttering $j(q)$ as a justification for some r C had the responsibility of justifying q on demand, in uttering instead $j(\emptyset(u))$ C defers that responsibility to A. B's constitutive recognition of A's inferential authority include his recognition of $\emptyset(u)$ as justifying q for C.

There must, of course, be some repeatable performance type which is used by members of the community as the expression of their recognition of some other member's having authorized the assertion of q by his utterance u . This will be some syntactically repeatable sentence-type in which another sentence can be em-

bedded, such as "A is committed to q". For a certain expression-type to play this role, community members must be disposed to token it only when they are disposed to recognize justifications and deferrals of justificatory responsibility in the ways we have just described. Further, the justification for such an attribution of authority-extended responsibility, should one be demanded, will be an anaphoric reference to the utterance u of A which prompts the attribution in the first place. The responsive dispositions associated with the use of the attributing locution must be such that performances of this type (the intension of the practice of recognizing authorizations) cohere with assertions, recognitions of justifications and other attributions of authority in the ways we describe.

The two factors which define recognition of a specific inferential licensing as codified in some repeatable expression-type, then, are first, recognition of certain new justifications involving anaphoric reference to the authorizing utterance, and second, recognition of deferral of further justificatory responsibility for the premissory assertions in such justifications which do make anaphoric appeal to an original assertion to the individual who issued that assertion. In setting out both of these factors, essential appeal is made to the recognition of a performance as a justification. In terms of this basic notion we can distinguish two levels of responsibility associated with assertion. First, there are sentences which an individual is disposed to assert, that is, explicitly undertake justificatory responsibility for and issue as inference-licenses. These may arise through inference from another assertion (in which case part of the justificatory responsibility may be deferred) or they may arise from responsive dispositions giving rise to language entries as in non-inferential reports. Let us call the set of such sentences which a community-member is prepared to assert at a time (determined with respect to some fixed but arbitrary standard of prompting) the basic repertoire of that individual. A superset of the basic repertoire will be the set of all sentences whose assertion the community (in this case, all those conversationally accessible) would take to have been inferentially licensed by the assertions the original individual is disposed to make i.e., would respond to with an expression of a type playing the recognitive role we have described in the last few paragraphs. We call this set the extended repertoire of the individual, in virtue of the extended responsibility which he has with regard to the sentences it contains. Such repertoires are to be our

analogues of systems of psychological states such as beliefs. They are social constructs in much the same way that community members are. In that case, the group of individuals whose recognitions would matter for A is determined by A's recognitions, but once these are settled, A has had his say, and those he recognizes determine his own membership or lack of it. Here, dispositions to utter assertion-candidates are entirely an individual matter, but the responsibilities one is taken thereby to incur are not. In both situations the individual proposes and the community disposes. In the next two sections, we will trace all semantic significance to this distinction between justificatory responsibilities explicitly undertaken by an individual and those attributed to him as implicitly undertaken in virtue of what he is prepared to assert.

First, however, we must complete our account of the systems of social practices which contain recognitions of performances as justifying others. Suppose someone has demanded of A a justification for his assertion u and A has responded with $j(\underline{u})$ a set of assertional utterances. For B to recognize $j(\underline{u})$ as a justification is for him other things being equal to attribute extended responsibility for the sentence-type of which u is a tokening (say, p) to every individual to whom B attributes at least extended responsibility for all the sentence-types of $j(\underline{u})$. Justifications must be portable across individuals, and create (defeasible) obligations for others (insofar as those others will be taken to have licensed the conclusion of such a justification by having licensed the premises). Of course, it is only those from whom B is conversationally accessible to whom he attributes such extended responsibility. A number of qualifications concerning this principle are in order. First of all, as stated the type-portability requirement assumes that all assertions of p by A license assertions of p by others. This would rule out justifications involving sentences like "I've just won the prize," assertions of which do not license type-repetitions. So we should restrict the principle as stated to those premissory assertions (members of $j(\underline{u})$) which are recognized (by all those conversationally accessible from A) as licensing type-identical re-assertions. Those sentences which are not so recognized form an antecedently definable class. We assume that each member of this class is recognized as licensing the assertion of some syntactically specifiable transform of the original type (changing "now" to "then" and "I" to "he" and so on). It is these transforms of the original premise-types attribution of extended responsibility for which induces recognition of extended responsibility for the

conclusion of the justification.

A second qualification is considerably more important. The extended responsibility induced by the presentation of a justification is defeasible by the performance of a counter-justification, comprising further assertions. The authority of reasons (our justificatory assertions must be amenable to modification by the advancement of further reasons. We will not assume any special characteristics of such counter-justifications except that they consist of assertions and that a counter-justification of a premise of a justification is always also (recognized to be) a counter-justification of the conclusion relative to that argument (that is, as erasing extended responsibility for the conclusion where such responsibility is already attributed for the premises). This last property captures the sense in which it is in virtue of the responsibility for the premises that one becomes responsible for the conclusion, and is therefore essential to the identification of a system of functionally responsibility-discharging performances as justifications. (Just as the universal responsibility for justifying performances on demand and the instituting of extended responsibility for the conclusion in others responsible for the premises by the performance of a justification set justification off from other responsibility-discharging performances like fulfilling promises). The categories of justificatory and counter-justificatory performances are not disjoint. A set of assertional performances may counter-justify the attribution of extended responsibility for one claim, and justify the assertion of another. Justification and counter-justification are functions such performances can have, according to their place in redistributing the responsibilities and authorizations for utterances of various expression-types according to a system of social practices.

With the provision for counter-justificatory performances, we complete the functional specification of the conditions under which such a system will be said to involve justificatory responsibilities (and hence assertions). This specification must distinguish justification from such other common forms of responsibility-fulfilling activity as promise-keeping, order-following. The features of our account which effect this distinction are: (i) the prior classification of performances into repeatable syntactic types, with both types and tokens respectively discriminable by the population, which can accordingly anaphorically refer to either; (ii) the universal appropriateness of demands for justification, themselves justifiable by anaphoric reference to the uttering of a sentence of

the type for which justification is demanded by the interlocutor from whom justification is demanded. (iii) performances which inferentially license further performances (recognition of this authority consisting in recognizing deferrals of justificatory responsibility in response to demands as in (ii) above), and thereby render their performers justificatorily responsible for sentence-types which they may not be disposed to assert, as well as those they are, (iv) that recognition of a successful justifying performance entails attributing extended responsibility for the sentence-type which was the conclusion to those who are accorded such responsibility for the premises, except if counter-justification is proffered, and (v) that any such attribution of extended responsibility can be defeated by counter-justification, where counter-justifying a premise entails counter-justifying the conclusion. Each of these conditions codifies some aspect of our ordinary practices of giving and asking for reasons. A system of social practices in which the relations of authority and responsibility it comprises satisfy these conditions may be called a justificatory system, and its performances are assertions.

Our account of the systems of practices within which utterances acquire assertional significance begins with the actual utterings and responsive dispositions of some cognitively structured community. In order to discuss the contents of these assertings, what is asserted in such performances, it will be useful for us to look at structures less tied to particular communities than these justificatory systems. Abstracting from the details of the responsive dispositions which give rise to them, we can capture the crucial conceptual structure of such systems by looking only at the basic and extended repertoires associated with each community member. The basic repertoire is the set of all sentences the individual is then prepared to assert whether in the original system the right so to assert was acquired by inference licensed by other assertions, or by non-inferential reports token-reflexively justifiable. The extended repertoire, a super-set of the basic repertoire, is the set of all those sentences which the community member would be held (by all of those conversationally accessible to him) to be justificatorily responsible for in virtue of the assertional dispositions codified in his basic repertoire. Such a set of basic and extended repertoires related by an accessibility relation will be called a conceptual idiom.¹¹ It follows that the same idiom may be exhibited by different justificatory systems. Note, however, that each such idiom is defined with respect to some defin-

ite syntactically specifiable set of sentences, so speakers of English and German could not share an idiom in this sense. It is in terms of these still rather particularized structures that we will define assertional contents or conceptual roles, which will allow us to consider cognitive relations between idioms defined on different domains of syntactically-specified sentence-types.

Before passing to that project, however, it is worth pausing to consider how such an idiom, abstracted from the justificatory system of one community, could be applied in interpreting the activity of some other community. To provide an interpretation of a community as acting according to the idiom will be to specify rules for associating with each individual in the target community at a time a certain repertoire (which we think of as having only the structure of its division between elements of the basic repertoire and sentences which are only in its extension) consistent with the conversational accessibility relations dictated by the political structure of that community. Congruence of accessibility aside, such an interpretation consists of two parts, each a specification of conditions under which one can come legitimately to evince a particular repertoire. First, some standard of prompting must be defined which gives content to the notion of disposition to utter on which attributions of basic repertoires are based. Such a standard had to be invoked in the original community in order to define the justificatory system from which the idiom is abstracted, and any interpretation according to that idiom must depend on some such further stipulation. Second, some specification must be given of the responsive dispositions which appropriately lead to the inclusion of non-inferential reports in a basic repertoire, prompted by a perceptible environing situation. The first part of the interpretation thus might specify what is to be meant by "assent" to a claim, while the second portion would say under what conditions one could appropriately assent to certain reporting sentences.

Following our account of the justificatory systems from which idioms are abstracted, the content of the interpretation of a community as exhibiting a particular idiom is that according to the rules the interpretation specified for maintaining and acquiring repertoires, each community member at each time is characterized by some repertoire admissible in the idiom. Not all the repertoires in the idiom need be actually occupied. Those that are not will be sets of possible dispositions (a double modality). The idiom then constrains, but does not determine, discourse according to it, since in actual justificatory systems anything

in one's current repertoire can be asserted (but of course one need not assert it), and similarly, at any time justification, counter-justification, and attributions of extended responsibility can be performed, but need not be. ^{11.5}

It should be pointed out finally that the attribution of an idiom to a community by such a two-part interpretation is consistent with an indefinite amount of deviation by actual participants from the dispositions appropriate according to the attributed idiom. For such deviations may be thought of as mistakes not justifiable according to the interpreted idiom. The interpreter, by so treating them, however, is committed to the in-principle explicability of all such errors in terms of a relatively small number of principles (e.g. concerning occasionally faulty perceptual mechanisms, or an individual history of defective linguistic training). In the sequel we will restrict our attention to the formal properties of conceptual idioms, and not return to their grounding in actual practice, whether by abstraction from such practice, or as interpretation of it.

III

To make out the claim that the systems of social practices we have described implicitly define assertion, we need to supplement that account of assertings with a story about the contents which are thereby asserted. Our starting point is Frege's discussion in the Begriffsschrift, where the distinction between force and content was first established. Frege writes:

...there are two ways in which the content of two judgements can differ; it may or it may not, be the case that all inferences that can be drawn from the first judgement when combined with certain other ones can always also be drawn from the second when combined with the same other judgments. The two propositions 'the Greeks defeated the Persians at Plataea' and the 'Persians were defeated by the Greeks at Plataea' differ in the former way...Now I call the part of the content which is the same in both the conceptual content (begriffliche Inhalt). Only this has significance for our symbolic language...In my formalized language...only that part of judgements which affects the possible inferences is taken into consideration. Whatever is needed for valid inference is fully expressed, what is not needed is for the most part not indicated either. (section 3)

Two features of this notion concern us. First, Frege identifies conceptual content with inferential role or potential. It is his project to find a notation which will allow us to express these precisely. Second, sentences have conceptual contents in virtue of facts about the appropriateness of material inferences involving them. The task of the logical apparatus of the conceptual notation which Frege goes on to develop is to make it possible to specify explicitly the conceptual contents which are implicit in a set of possible inferences which are presupposed when Frege's logician comes on the scene. The task of logic is thus set as an expressive one, to codify just those aspects of sentences which affect their inferential potential in some pre-existing system.

We will derive conceptual contents from the systems of practices of inference, justification, and assertion described above. Following the Fregean philosophy of logic, we do so by introducing formal logical concepts as codifications of material inferential practices. First we show how conditionals can be introduced into a system of practices of using basic sentences, so as to state explicitly the inference license which the assertion of one sentence which becomes the antecedent of the conditional) can issue for the assertion of another (the consequent

of the conditional). With conditionals constructed so as to capture formally the material inferential potential of basic sentences, we then show how conceptual contents expressed in terms of such conditionals can be associated with basic sentences on the model of the introduction and elimination rules for compound sentence forms like the conditional.

Before doing so, however, we must take note of a complication which arises when the conceptual contents of individual sentences are taken as codifying inferential potentials of those same sentences. The problem is the relativity to context of such material inference from a given sentence, registered by Frege in the passage quoted above when he requires for the matching of inferential roles that "...all inferences that can be drawn from the first judgement when combined with certain other ones can always also be drawn from the second when combined with the same other judgements" (emphasis added). We cannot in general talk about "the consequences" of a claim (for instance, that the moon is made of green cheese) without somehow specifying a context of other claims against the background of which such consequences can be drawn. (Can we use what we know about the mammalian origins of cheese and take as a consequence that at one time the moon was made of milk, for instance?) Quine, in "Two Dogmas", may be seen as arguing against the possibility of an atomistic theory of meaning (e.g. one which assigns to every sentence its 'conceptual content') that such meanings must at least determine the inferential roles of sentences, and that the roles of each sentence in a "web of belief" depends on what other sentences inhabit the same web. In particular, whether anything counts as evidence for or against a certain claim, (where it is essential to attributions of analyticity to a sentence that nothing should have this inferential role with respect to it) depends on what other sentences are held concurrently. Given any sentence, there will be some possible doxastic contexts in which nothing counts as evidence for or against that sentence, and given any second sentence there will be some webs in which the second counts as evidence for the first, and some where it counts as evidence against the first, where what 'web of belief' is considered determines what other sentences are available as auxiliary hypotheses for inferences. Accepting the general Fregean line that meanings as theoretical constructs are postulated to express inferential potentials, Quine reminds us of basic facts about our inferential practices (such as those expressed in Duhem's thesis) to impugn the comprehensibility of assignments of conceptual role to individual sentences, unre-

lativized to some doxastic context. Conceptual roles can only be specified relative to a set of other sentences which are all and only those which can be used as auxiliary hypotheses, that is, as Quine puts it, at the level of whole theories-cum-languages, not at the level of individual sentences. We will argue below that the relativity of inference to doxastic context can be reconciled with the notion of sentential conceptual roles, showing that Quine's inference from the holistic character of our inferential practices to the impossibility of their codification in conceptual roles matched up with individual sentences overlooks important possibilities.

The first step in our account of the semantic contents or conceptual roles sentences acquire in virtue of being used according to the practices expressed in some idiom is the introduction of some logical vocabulary. We understand the inference-licensing function of assertion by our model of justificatory systems of social practices. We will introduce the conditional as a compound sentence-form constructed out of the basic sentences on which some idiom is defined. The conceptual content of the conditionals will be stipulated; a sentence of the form $p \rightarrow q$ is to have the content as inference-license of a statement of the appropriateness of an inference from the assertion of p to the assertion of q . Various formal inferential connections between such conditional sentences will then be elicited. For these formal principles to comprise a logic is for them to make possible the explicit formal codification of the material inferential and justificatory practices of some conceptual idiom. This is the task Frege sets for logic in his Begriffsschrift--although in that work he succeeded only in completely codifying the formal inferences involving his logical constructions, his discussion makes clear that the ultimate criterion of adequacy for his conceptual notation is its capacity to express as explicitly and precisely the contextual material inferences which define the conceptual roles of non-logical sentences. Once the conditional has been introduced as codifying the consequence relation implicit in material inferential practice, and its formal logical properties have been presented, we will use such conditionals both as models for the conceptual roles of non-logical sentences (which will have analogues of introduction and elimination rules, and will be given content as licensing inferences from their circumstances of appropriate application to the consequences of such application) and as tools for making those roles explicit.

We may think of the relation between basic and extended repertoires in a

conceptual idiom as defining a consequence function on admissible sets of sentences. For the extended repertoire, recall, comprises just those sentences which an individual would socially be held responsible for (in the sense that the relevant community members would recognize anaphoric deference of justificatory responsibility for claims of those types to that individual) in virtue of the dispositions that individual displays explicitly to undertake such responsibility for the sentences in his basic repertoire. The extended repertoire consists of those claims the community takes him to be committed to by being prepared to assert the claims in his basic repertoire. These community practices thus induce a consequence function which takes any admissible basic repertoire and assigns to it its consequence extension. The function only represents the consequences of individual sentences relative to some context, since we know what the consequences are of p together with all the other sentences in any basic repertoire containing p, but so far have no handle on which of these various consequences might "belong" to p. Thus we have just the sort of material inferential relations Frege presupposes when he talks of the inferences which can be drawn from a given judgement "when combined with certain other ones" in the passage quoted above.¹² The idiom also expresses a material consistency relation, since in general not all sets of sentences will be (recognized as) basic repertoires. The sets which are not idiomatically admissible repertoires are sets of sentences which one cannot have the right simultaneously to be disposed to assert, according to the practices (e.g. reporting practices) of the community from which the idiom is abstracted. The final component of a conceptual idiom as we have defined it is the conversational accessibility relation between repertoires. We may represent such idioms by triples of the form $(\mathcal{R}, \underline{c}, J)$, where \mathcal{R} is a set of sets of sentences (the basic repertoires), \underline{c} is a function from basic repertoires to the supersets which are their consequence-extensions, and J is a function which assigns to each repertoire the class of accessible repertoires.

Given such an idiom defined on a set of non-logical sentences, we will add conditional sentences $\underline{p} \rightarrow \underline{q}$ to each of the consequence-extended repertoires in which, intuitively, p is inferentially sufficient for q, in such a way that the newly minted sentences have the standard inferential consequences of conditionals and so that this formal swelling of the original repertoires is inferentially conservative, that is does not permit any material inferences, which were not already permitted in the original idiom. Once again taking our lead from Frege, we

may say that p is actually (inferentially) stronger than q at basic repertoire S just in case the consequences of q taken together with S are also all consequences of p taken together with the same context, that is, if $c(S \cup \{q\}) \subseteq c(S \cup \{p\})$. Recalling the constitutive role of recognitions by accessible community members in determining consequence relations, we may further define p as juridically (inferentially) stronger than q at some repertoire R just in case p is actually stronger than q at every repertoire S accessible from R . This natural modal version of inferential sufficiency will be our semantic introduction rule for conditional sentences, that is, we will add $p \rightarrow q$ to the consequence-extension of a repertoire R just in case for all $S \in J(R)$: $c(S \cup q) \subseteq c(S \cup p)$, where we have written " $S \cup q$ " for $S \cup \{q\}$. The conditional thus has a particular content in the context of a given repertoire, a content determined by the inferential roles played by its antecedent and consequent.

The formal implementation of this idea requires a little care. We must show that the important formal properties of idioms are preserved by the introduction of conditionals, and that the conditionals so introduced have appropriate properties. In order to permit sentences with more than one arrow in them, we must swell the basic idiom with conditionals first, and then iterate the process adding conditionals which can have first-order conditionals as antecedents or consequents, and so on, showing that the relevant properties of conceptual idioms are preserved at each stage. Our procedure is this. Starting with a basic idiom $I_0 = (R_0, c_0, J_0)$, we define a new idiom I_1 with repertoires defined not just over the original set of non-logical sentences, but also containing first order conditionals, as well as consequence and accessibility relations between them. The same procedure is repeated, and eventually we collect all the results in the formally expanded idiom $I = (R, c, J)$, defined by:

$$R = \bigcup_{k=0}^{\infty} R_k$$

$$c(R) = \bigcup_{k=n}^{\infty} c_k(R) \quad \text{and}$$

$$J(R) = \bigcup_{k=n}^{\infty} J_k(R), \quad \text{where } R \in R \text{ and } n \text{ is the}$$

index of the first R_k such that $R \in R_k$.

The properties of conceptual idioms which must be preserved at each stage in this construction are these. First is the extension condition, that for any admissible repertoire R , $R \subseteq c(R)$. The motivation for this condition is that the consequence extension $c(R)$ of R is to represent those claims one is taken to be

committed to in virtue of being prepared explicitly to take responsibility for the members of R , and certainly one has committed oneself to the claim one asserts, and licenses the trivial inference which is re-assertion justified by anaphoric deferral to one's original performance. Second of the properties of conceptual idioms which we make use of is the interpolation condition, which specifies that any basic repertoire R which can be exhibited as the result of adding to some other repertoire S sentences each of which is contained in the consequence extension of S , has as its consequence extension $c(R)$ just the set $c(S)$. That is, for all repertoires R , if there is a repertoire S such that $S \subseteq R \subseteq c(S)$, then $c(R) = c(S)$. The reason for imposing this condition is that an individual exhibiting the assertional dispositions codified in the basic repertoire S could come to be characterized by the dispositions codified in R simply by coming to acknowledge explicitly some of the consequences of what he already is prepared to acknowledge explicitly. Doing so should not alter the justificatory commitments which the relevant members of the community attribute to him. Assertion of something one is already held responsible for in virtue of previous assertions does not change the responsibilities these performances undertook.

The interpolation condition allows us to be liberal in populating idioms with repertoires, since if S is admissible (materially consistent) according to an idiom, so should be any R 'between' S and its consequence extension. So we may as well just stipulate that all such sets are included as repertoires in any conceptual idiom, that is that for all sets of sentences S , if there is an $R \in \mathcal{R}$ such that $R \subseteq S \subseteq c(R)$, $S \in \mathcal{R}$. The interpolation property ensures that the consequence function will be defined on all of these sets, so adding them won't make c a partial function, and we can stipulate, for the same reasons already advanced, that the accessibility relations of these interpolated repertoires S will be the same as those of the basic repertoires R which generate them (we'll say a bit more about this point below). The idempotence of the consequence function, that for all $R \in \mathcal{R}$, $c(c(R)) = c(R)$, is a consequence of the interpolation property. Of course this is a desirable circumstance, since we want idempotence in the relation which is interpreted as the closure under material inference (as constituted by social attributions of justificatory responsibility) of admissible basic repertoires. So the domain \mathcal{R} of any idiom comprises a set of basic repertoires, and for every repertoire (basic or not) in \mathcal{R} , \mathcal{R} contains also its consequence extension and all interpolations between that repertoire and its conse-

quence extension.

It is worth pointing out that we do not require of pairs of repertoires $R, R' \in \mathcal{R}$ that if $R \subseteq R'$ then $c(R) \subseteq c(R')$. The consequence relation is contextual, in that a change in the total evidence which merely adds to that evidence may entail the denial of some claims which were consequences of the evidential subset. Allowing such a possibility is crucial for codifying material inferential practices, which are almost always defeasible by the introduction of some additional auxiliary hypothesis or other. Of course doing so makes the construction of conditionals more difficult, since it requires a conditional for which the standard principle of antecedent-strengthening (namely that if $p \rightarrow q$, then $p \& r \rightarrow q$ for any r) fails, as it does for the everyday sense of 'if...then___' according to which both 'If I strike this match it will light,' and 'If I strike this match and I am under water, it will not light,' can be true and justified. Denying monotonicity (that if $R \subseteq R'$, then $c(R) \subseteq c(R')$) forces our logic to take account of the relativity of material inference to total evidence at the outset, with relativity to context made an explicit part of the formalism instead of leaving that phenomenon to the embarrassed care of ceteris paribus clauses because standard conditionals capture only formal inference, which is not context-sensitive.

The formal properties of the accessibility relation of which we must keep track are generated by our specification that in conceptual idioms only differences in repertoire could make differences in accessibility, together with our treatment of interpolated repertoires. We require that if $c(S) = c(S')$, then a) if $S \in J(R)$ for some $R \in \mathcal{R}$, then $S' \in J(R)$, and b) if $R \in J(S)$, then $R \in J(S')$, that is, that consequence-extensions determine accessibility relations. We must also be sure that algebraic properties of the accessibility relation, reflexivity, symmetry, and transitivity, are preserved as we construct the formal expansion of a basic idiom.

We assume that we have a basic idiom $(\mathcal{R}_0, c_0, J_0)$ meeting these conditions, comprising repertoires defined on sentences containing no non-logical vocabulary. Given any $(\mathcal{R}_k, c_k, J_k)$, for instance the basis where $k=0$, we construct $(\mathcal{R}_{k+1}, c_{k+1}, J_{k+1})$ by introducing conditionals. The construction is in five steps:

1) For all $R \in \mathcal{R}_k$, $c_{k+1}(R)$ is the smallest set (by inclusion), such that:

- i) $c_k(R) \subseteq c_{k+1}(R)$ and
 ii) $p \rightarrow q \in c_{k+1}(R)$ if for all $S \in \mathcal{R}_k$

if $S \in J_k(R)$ and $(S+p) \in \mathcal{R}_k$ then $c_k(S+q) \subseteq c_k(S+p)$.

(ii) is the conditional-introduction rule discussed above.

(i) simply preserves the extension property.

2) \mathcal{R}_{k+1} is the smallest set such that:

- i) $\mathcal{R}_k \subseteq \mathcal{R}_{k+1}$ and

ii) For any set of sentences S , if $\exists R \in \mathcal{R}_k (R \subseteq S \subseteq c_{k+1}(R))$,

then $S \in \mathcal{R}_{k+1}$.

Condition (i) ensures that if a repertoire is admissible at any level in the construction, it will be admissible at all higher levels. Condition (ii) just makes sure that interpolation will be preserved.

3) For all $R \in \mathcal{R}_{k+1}$, $c_{k+1}(R) = c_{k+1}(R \cap (\cup \mathcal{R}_k))$, where $\cup \mathcal{R}_k$ is the set of all sentences which are elements of elements of \mathcal{R}_k . This step in the construction provides consequence extensions for the new repertoires constructed in (2), and ensures that no further sentences constructed at lower levels are added as consequences at this stage (recall our conservatism desideratum).

4) For all $R \in \mathcal{R}_k$, $J_{k+1}(R)$ is the smallest set such that:

- i) $J_k(R) \subseteq J_{k+1}(R)$, and

ii) For all $S \in J_{k+1}(R)$, For all $S' \in \mathcal{R}_{k+1}$, if $c_{k+1}(S') = c_{k+1}(S)$, then $S' \in J_{k+1}(R)$.

(i) preserves previously constructed accessibility relations, and (ii) guarantees that property (a) of accessibility relations will hold at each level of the construction.

5) For all $R \in \mathcal{R}_{k+1}$, $J_{k+1}(R) = J_{k+1}(R \cap (\cup \mathcal{R}_k))$.

This condition constructs accessibility relations for the new repertoires added at each constructive level, in keeping with our remarks about the interpolation property. Property (b) of accessibility relations is preserved at each stage, since it is clear by construction that if

$J_k(R \cap (\cup \mathcal{R}_k)) = J_k(R' \cap (\cup \mathcal{R}_k))$, then $J_{k+1}(R) = J_{k+1}(R')$. It is also clear from the construction of J_{k+1} that if J is reflexive, symmetric, or transitive, so is each J_k , and hence J . Notice that in general, if $c_k(R) = c_k(R')$, then $c_{k+1}(R) = c_{k+1}(R')$.

We are now in a position to investigate the logic of the arrow which this formal, non-substantive expansion of the basic idiom induces. To do so, we look at the sentences which are idiomatically valid, in that every repertoire in the formally expanded idiom contains these sentences in its consequence extension. First, and as an example, we show that if \underline{p} is in some consequence-extended repertoire, and $\underline{p} \rightarrow \underline{q}$ is also in that repertoire, then so is \underline{q} , that is, that modus ponens is supported by the arrow:

Theorem 1 - IELIM Let $k+1$ be the lowest constructive level such that $\underline{p} \in c_{k+1}(R)$ and $\underline{p} \rightarrow \underline{q} \in c_{k+1}(R)$ for any R . Then by (lii) For all $S \in \mathcal{R}_k$, if $S \in J_k(R)$ and $S+\underline{p} \in \mathcal{R}_k$, then $c_k(S+\underline{q}) \subseteq c_k(S+\underline{p})$. Assuming that J_0 (and hence J) is reflexive, we can let $S=R$, since $R \in J_k(R)$ and $R+\underline{p} \in \mathcal{R}_k$ (since $\underline{p} \in c_k(R)$ by interpolation, (2ii), and that \underline{p} has at most one fewer arrows than $\underline{p} \rightarrow \underline{q}$). So $c_k(R+\underline{q}) \subseteq c_k(R+\underline{p})$, and by interpolation $c_k(R+\underline{p})=c_k(R)$. By the extension property, then, $R+\underline{q} \subseteq c_k(R+\underline{q})$, so by the transitivity of the subset relation, $\underline{q} \in c_k(R)$, and so by (li) $\underline{q} \in c_{k+1}(R)$. QED.

Our general account of the introduction of the conditional specified when one was entitled to assert one (based entirely on formal properties of the repertoire one exhibits), and this condition tells us what the significance of these conditionals is (and, incidentally, established our right to call the arrow we have constructed a "conditional"). Proofs of further theorems of the logic of conditionals our construction induces may be found in Appendix I. As the discussion in that appendix illustrates, the most unusual feature of the resulting logic is its two-class structure, treating conditionals whose antecedents or consequences are other conditionals rather differently from the way in which it treats conditionals involving only basic sentences. This feature is a direct consequence of the introduction of first-order conditionals based on material inferential circumstances of the repertoire in question, and higher-order conditionals according to purely formal, materially conservative criteria. Thus it is obvious from inspection of the five steps of our construction of the hierarchy of conditionals that the complement of basic sentences in a consequence extended repertoire is never altered during that construction, and that the novel repertoires introduced always have first-order restrictions which are elements of the original set \mathcal{R}_0 . Higher-order conditionals, of course, are what are added to the original idiom, and as is clear from Appendix I, those conditionals obey a standard modal logic. The principles governing conditionals with basic sentences as antecedents or con-

sequents, however, are those of the pure implicational fragment of Belnap and Anderson's system EI of entailment.

We may view the conditionals which end up included in the consequence extensions of formally expanded repertoires as partially ordering all of the sentences of the (syntactically specified) language. Since according to our introduction rule, a repertoire will contain conditionals whose antecedents and consequents are not contained in that (extended) repertoire, the ordering so induced is not limited to the sentences of the repertoire from which the ordering conditionals are drawn. Although the conditional induces an appropriately transitive and reflexive relation on the sentences of the language, the ordering will not be total (since for some p, q and $R \in \mathcal{R}$, it may be that neither $p \rightarrow q$ nor $q \rightarrow p$ is in $c(R)$), and it will not be complete, in that sentences appearing only in inaccessible repertoires will have only trivial implication relations (e.g. $p \rightarrow p$).¹³ The conditionals which do not have antecedents in $c(R)$ are counterfactual with respect to R . These are of three kinds i) those taken true by the theory codified in the repertoire, that is, counterfactuals in $c(R)$, ii) those taken not to be true, i.e. conditionals not in $c(R)$ but on which R induces non-trivial entailments, and iii) inaccessible counterfactuals, assigned no significance by the expanded repertoire (e.g. "If the number seventeen were a dry, well-made match..."; an antecedent generating counterfactuals which, with respect to a certain set of beliefs or repertoire simply makes no sense.). Entailment relations between counterfactuals of the first two kinds and between each of them and base sentences will be underwritten by the induced partial ordering, all depending on the original material inferential practices involving only base sentences.^{13.5}

The repertoire which induces such a partial ordering by its conditionals will then be a distinguished subset of the sentences it orders, one which Theorem 1 assures us is deductively closed under modus ponens. Each repertoire is in short a theory or set of beliefs, embedded in a larger linguistic structure defining the implications of the sentences in that theory. Not only do different repertoires codify different theories, but they assign different significances to syntactically type-identical sentences of those theories, in that p as an element of $c(R)$ may have one set of inferential consequences, and as an element of $c(R')$ have a different set of consequences. The repertoires ordered by their indigenous implication relations thus deserve to be called "webs of belief" in Quine's sense, as the smallest units of analysis within which sentences have significance. The idiom, comprising all of these repertorial structures of implicational sig-

nificance and embedded belief, is not a set of meanings common and antecedent to the repertoires, but is the structure within which each such web of belief is a linguistic perspective made possible by a justificatory system of social practices.^{13.75}

To see the importance of this last point, we need only notice how the possibility of genuine communication can come to seem mysterious in the context of a Quinean holism about meanings of sentences. For it is natural to take as a criterion of such communication that some sentence, uttered as having a certain significance, is taken by its audience to have that same significance. Yet if the significance of any sentence depends on all the other beliefs held by the one who entertains it, communication on this model will be impossible for individuals just insofar as they have different beliefs. Since communication is important just insofar as the interlocutors don't have common beliefs, this is a distressing result, and might be taken as a reductio ad absurdum of the holistic view. Indeed, although the discussion has largely been couched in terms of the diachronic communication (in "commensurable" terms) required between those who espouse different scientific theories if we are to make sense of empirical inquiry as progressive, I believe that the challenge of accounting for non-defective communication according to holistic views such as Quine's which relativize meaning to contexts of belief has been a primary motive for the attempt by people like Putnam and Field to develop causal theories of reference. On the account so envisaged, although my religious beliefs may give an idiosyncratic implicational significance to your innocent report "The sun has gone behind that thundercloud", shared causal factors ensure that we both are talking about the sun and the same thundercloud. As we have told the story, however, one begins with a set of social practices of producing assertional utterances and making inferences from them and then extracts significances from this communicational context. Only the syntactically identified sentences which are exchanged need be common to the various conversational interlocutors. The systematic variation of the significance of those sentences from one individual to another expressed in a formally expanded idiom then exactly answers to whatever communication is going on in the initial set of practices. The possibility of communication consists in just the kind of coordination of significances across repertoires codified in a formally expanded idiom.

Before discussing the detailed representation by means of conditionals of these 'significances' in the next section, we should show that the condition-

als we have constructed out of consequence extensions of repertoires generated by attributions of inferential licensings can serve as social-practical as well as merely formal expressions of recognitively constituted inferential relations. We have described the practical origins and effects of elements of extended repertoires which are first-order sentences of the language, in terms of attributions and undertakings of justificatory responsibility and the issuing and recognition of inferential authority. What, in these terms, should we take to be the significance of the persence in such a repertoire of a conditional $p \rightarrow q$? The presence of such a conditional in the formally expanded consequence extension of the repertoire exhibited by an individual should signify, first, that that individual recognizes others who are prepared to assert p as licensing the inference to q , and second, that he recognizes the assertion of p as justifying the assertion of q . The criterion of adequacy for assigning such a social-practical function to the conditional is that if $p \rightarrow q \in c(S)$ for every admissible S accessible from some admissible repertoire R , and R contains p , R should contain q as well, since all those whose recognitions are relevant accept $p \rightarrow q$. But notice now that since we assume that the accessibility relation is reflexive, each S is accessible from itself, so by our introduction rule for the arrow (lii) $c(S+q) \subseteq c(S+p)$ whenever $S+p \in R$. But since this holds for all $S \in J(R)$, this means that $p \rightarrow q \in c(R)$. Part of our hypothesis was that $p \in c(R)$, so by the detachment property proven in Theorem 1, $q \in c(R)$. So if all those recognized by the individual exhibiting R are responsible for the conditional $p \rightarrow q$ and $p \in c(R)$, then $q \in c(R)$, which means that $p \rightarrow q$ plays the proper role as codifying the recognition of inferential licensing and appropriate justification of q by p .

Finally, we state a more general condition under which the arrow we have defined will be a practically complete expression of the a justificatory system: (PC) $\forall R \in \mathcal{R}, \forall q, q \in (c(R)-R)$ iff $\forall S \in J(R), \exists x \in R (x \rightarrow q \in c(S))$. If this condition holds, then every attribution of extended responsibility for q by some accessible individual will be represented and licensed by the presence in the expanded repertoire of that individual of an appropriate conditional. This formulation allows that these licensing conditionals may vary from accessible repertoire to accessible repertoire. Systems meeting this condition represent a cognitive ideal, since in it a justification of every utterance of a sentence in one's extended repertoire can be justified by means of conditionals, and every

such justification can be so justified, and so on. The claims we will make in the next section about the conceptual priority of a constructed conditional for a theory of meaning are most plausible for a practically complete justificatory system, but are not intended to be restricted to that ideal case.

IV

Our strategy now is to use the conditionals we have constructed to develop precise representations of the conceptual contents sentences acquire in virtue of playing a material inferential role in some justificatory system. The most sophisticated use of the notion of a conceptual role has been made by Sellars, who in Science and Metaphysics and elsewhere develops a theory of meaning couched in terms of dot-quoted expressions, where such dot-quotation of an expression results in a term referring to the conceptual (inferential-justificatory) role of that expression.¹⁴ Sellars takes dot-quotation and conceptual roles as primitives, and is concerned to explicate such phenomena as indirect discourse, translation, and reference to abstract entities in terms of them. To put these insights to best use, it is necessary to explicate the notion of conceptual role on which this approach centers. According to the present view, it is the defining task of a logic or logical construction that it make possible the explicit codification in a conceptual role of what is implicit in the inferential and justificatory employment of an expression. In this section we show how conceptual roles in Frege's and Sellars' sense can be expressed, using the conditionals of our formal logic not only as the means of expression of roles, but also as providing the model according to which we understand such roles.

The key to this line of thought is the observation that the only sentences whose roles we understand explicitly are the conditionals. We understand them because we constructed them, stipulating their introduction conditions, and deriving the consequences of such introduction (the validity of detachment). We propose to generalize this clear case, and conceive the mastery of the use of an expression which one must exhibit in order to properly be said to understand it ("grasp" its conceptual role) as consisting of two parts, knowing when one is entitled to apply the expression, and knowing what the appropriate consequences of such application are (what justifies using the expression, and what inferences one licenses by so doing). Applying the expression is thus assimilated to performing an inference from the circumstances of appropriate application of the expression to the consequences of its application. Although in this paper we are concerned only with the first two levels, it is important to notice that this model of conceptual roles applies not only to connectives (whose introduction and elimination rules correspond to circumstances and consequences of application respectively) and sentences, but to terms as well. Thus Dummett analyzes our reluctance to use

certain terms, such as "nigger", as proper when we would not be willing explicitly to license the inference from its circumstances or criteria of application (here, the color of someone's skin) to the consequences of such application (e.g., the social negligibility of the individual).¹⁵ Using the term is licensing such an inference, and so the term should not be used nor its use acquiesced in by those who do not accept the inference.

On this model, suggested by the later Carnap's use of partial reduction forms, the conceptual role of any expression is the pair of its circumstances of appropriate application and the consequences of such application, that is, of its (individually) sufficient conditions and of its (jointly) necessary conditions. The application of that expression is to be thought of as an inference from the former to the latter. Assertion thus becomes a limiting case of inference. Such a role will be relative to a repertoire, and may be represented explicitly according to: Definition-The conceptual role of p with respect to basic repertoire R , written

$$/p/R =_{df} \langle \{x: x \rightarrow p \in c(R)\}, \{y: p \rightarrow y \in c(R)\} \rangle ,$$

whose first element, the circumstances of application, we write $1(/p/R)$, and whose second element, the consequences of application, we write $2(/p/R)$.

This definition captures the sense in which conditionals are used as tools in the expression of conceptual roles (our precise version of significances or contents).

More must be said, however, about the ramifications of taking conditionals to be the models for the conceptual roles of basic sentences, inasmuch as our strategy has been to construct a conditional as stating explicitly (as a license) what is implicit in an inference from its antecedent to its consequent, and then to assimilate the content of basic statements to the model of these constructed conditional statements. In general, one might think that it was incoherent or circular to define the contents of the categorical sentences of an idiom in terms of the contents of hypothetical sentences of that idiom (for where do those contents come from if not in turn from the categoricals?). Our construction avoids this worry, since we define conditionals in terms of the contents of basic sentences only in the sense in which those contents are implicit in the formal inferential practices which are the use of the basic sentences. The explicit contents thus constructed for conditionals are codified in the valid sentences of the arrow logic canvassed in Appendix I, and are used in turn to express explicitly the contents originally implicit

in the use of the basic sentences. So there is no circularity in taking the contents of conditionals as models for those of basic sentences involved, merely the introduction of a formal auxiliary apparatus of conditionals which add no material content to the basic sentences they are defined on, merely performing an expressive function (the criterion of formal or logical theoretical constructs). Nor is there anything peculiar about taking a sub-class of sentences as the paradigms to which all others are assimilated in a theory of meaning. Frege, for instance, treats all sentences as implicit identity statements (involving names of the True or the False), because of the particular way in which the issue of informativeness and the possibility of communication is raised and addressed in "On Sense and Reference". Thus Frege constructs a theory of meaning based on terms explicated with the logical device of identity, where we base our account on sentences explicated by means of the logical device of conditionals. We attempt to give a direct account of saying and what is said which does not appeal to naming and what is named. This is the essential difference between conceptual role semantics inspired by the sort of concerns articulated by the later Wittgenstein, and referential semantics inspired by Frege.

As Dummett¹⁶ points out, the later Frege broke from previous logicians in treating logic not as the study of inference, but of a special kind of truth (cf. Frege's "The Thought"). This view seems to have been motivated by his presentation of logic as an axiomatic system, where some truths are stipulated and others truths are derived from them by a minimum of purely formal inferential principles. The philosophical critique in terms of linguistic practice of the distinction between meaning-constitutive stipulated truths and empirically discovered truths, together with Gentzen's achievement of parity of formal power between proof-theoretic methods of studying consequence relations and the truth-oriented methods epitomized by matrix interpretations (which he achieved by complicating inference rules so as to allow the discharging of hypotheses) require us to reassess the relations of explanatory priority between the notions of inference and truth. One of Frege's achievements is his formulation of the principle of semantic explanation, according to which the appropriateness of a form of inference is to be accounted for by showing that it never leads from true premises to conclusions which are not true. The usual way in which to exploit this principle is to begin with an account of truth (typically in representational or referential terms) and partition a space of abstractly possible inferences and forms of inference into those

which are appropriate and those which are not appropriate according to the semantic principle, as Frege does in the Begriffsschrift. Our approach in effect reverses this order of explanation, beginning analysis with a set of appropriate inferences and explaining semantic interpretants, including truth-values, in terms of them.

This may seem to be an absurd project. For why should we think that knowing all the facts of the form "From p it is appropriate to infer q" would or should enable us to establish the truth values of all of the antecedents and consequents involved in these inferential relations? Putting the matter a little more precisely, if we express all the relevant inferential facts in the form of hypothetical sentences in the conditional form "if p then q", surely we do not want it to turn out that the truth-values of all such hypothetical sentences determined the truth-values of all the categorical sentences out of which they are constructed? The hypothetical sentences ought only to establish possible worlds by determining compossible sets of sentences. Just as it is implausible to take what is possible as determining what is actual, so it is implausible to take the totality of conditional truths as determining the totality of unconditional truths. Indeed, the possession by a formal system of this semantic property would be a strong reason to take its conditional as not a reasonable rendering of the English hypothetical construction "if...then". Embarrassingly enough, the standard truth-functional (mis-named "material") conditional which Frege employs has just this property, namely that if the truth-values of all of the conditionals of the language are settled, then the truth-values of all the sentences of the language are settled. This is proven in Appendix II. Our genuine conditional, introduced as codifying a set of non-formal inferences, will not have this undesirable property. Concern with the truth-values of categorical sentences arises in the context of our analysis of inference as explicitly codified in conditionals more plausibly and according to quite different principles than the sort of "account" of the categorical sense of "taking-true" relevant to assertion in terms of the hypothetical sense relevant to inference which is detailed in Appendix II. We avoid that result by taking the principle that appropriate inference should never lead from true premises to conclusions which are not true as a necessary, but not sufficient condition for appropriateness of inference. The truth-functional conditional results from taking the principle to provide sufficient conditions as well.

To see more clearly the semantic relations between inference and truth-values, we pursue briefly the significance of our opening observation that semantic content will be the common contribution the sentence p makes to assertions of it as a free-standing utterance and assertions of conditionals containing it. Taking Frege's semantic explanatory principle as a necessary condition on an account of inferential relations settles that the primary semantic notion will be whatever it is that is preserved by appropriate inferences. Frege calls this 'truth', but abstractly there are other properties which could also play this role (e.g., justificatory responsibility) and there are good reasons to expect an adequate semantic theory to account as well for the preservation of 'relevance' of some kind by appropriate inferences.¹⁷ This primary semantic notion, however, pertains only to the use of a sentence as a free-standing assertive utterance. A full notion of sentential content must specify as well the role a sentence has as a component in other, compound, sentences, paradigmatically in conditionals. It cannot be determined a priori that these two roles coincide.¹⁸ If with Frege we take the first semantic property to be a truth-value either possessed or not by any sentence, then the assumption that the second or componential notion coincides with the first results in classic two-valued truth-functional logic.

That the two varieties of semantic interpretant (one answering to role in inference, the other to role in sentential compounding) can diverge is shown by the general form of semantic matrices which grew up around Frege's initial suggestions. An example will make the point clear. Consider a standard three-valued conditional logic specified by a matrix

\Rightarrow	1	2	3
*1	1	2	3
2	1	2	2
3	1	1	1

This connective is not presented as interesting in its own right, but merely as an example of a conditional which satisfies Frege's semantic explanatory principle, if modus ponens is the rule of inference. The '*' indicates the designatedness of a multivalued. In this case 1 is the designated value. Designatedness is what is stipulated to be preserved in inference, it is the formal representative of the primary semantic notion described above. Notice now that the two undesigned multivalued 2 and 3 make different contributions to the designatedness of compounds containing them. Thus consider the conditional statement $(p \Rightarrow q) \Rightarrow r$. Suppose

the multivaluedness of p is 1 and that of r is 2 . Then if the value of q is 3 , by the table above, $p \Rightarrow q$ has the value 3 , and the value of $(p \Rightarrow q) \Rightarrow r$ will be 1 . If, on the other hand, q is given the value 2 , the other undesigned value, then the value of $p \Rightarrow q$ will be 2 and that of $(p \Rightarrow q) \Rightarrow r$ will also be 2 . Thus substituting one undesigned multivaluedness for another alters the designatedness of the compound sentence.

Thus we see that many-valued semantics requires the assignment to each sentence of two different sorts of semantic interpretation: a designatedness value indicating possession or lack by a sentence used as a free-standing utterance of the property which appropriate inference must preserve, and a multivaluedness codifying the contribution the sentence makes to the designatedness value of compound sentences containing it, according to the principle M:

M) Two sentences have the same multivaluedness iff they are intersubstitutable salva designatedness value in every sort of compound sentence (including the trivial compound which consists just of the free-standing sentence itself).

More generally, such a semantics is a matrix, consisting of a set A of multivaluednesses, a subset D of A , the designated multivaluednesses, and for each kind of n -ary sentential compounding (e.g. a connective) a function from A^n into A (corresponding to the multivaluedness-table of the connective, as in our example). An interpretation of a language L according to such a matrix is a mapping of the primitive propositions of L into elements of A . A valuation of L is then induced by extending this mapping to compound sentences using the connective functions in the obvious way. The matrix verifies a sentence p of L just in case for every valuation the multivaluedness assigned to p is designated. A matrix is characteristic for a logic if it verifies just the theorems of that logic. Lindenbaum showed that every logic has a characteristic matrix, namely the one gotten by taking the set of multivaluednesses to be classes of inferentially equivalent sentences, and the designated multivaluednesses to be the theorems of the logic in question.

We are now in a position to notice that a repertoire, together with the partial ordering induced on the sentences of a repertoire by the conditionals contained in its formally expanded consequence extension constitute such a Lindenbaum matrix. The multivaluednesses of the matrix are just equivalence classes of sentences of the language under the relation: $p \sim q$ iff $(p \rightarrow q \in c(R) \text{ and } q \rightarrow p \in c(R))$. The designated multivaluednesses are then those sentences (strictly classes of them) p for which $p \in c(R)$. The designated conditionals, then determine the function

which is assigned by that repertoire to the arrow. Theorem 1 above shows that modus ponens preserves designatedness, that is membership in the extended repertoire. Or, to put the same point another way, that result shows that our constructed conditional satisfies Frege's semantic explanatory principle when membership in a repertoire is taken as the prime semantic notion, and social practice determines an antecedent class of appropriate material inferences. The formally expanded repertoire thus is, in a precise sense, the characteristic semantic matrix not for a logic or set of formal inferences, but for a set of material inferences, namely those appropriate from the point of view of the individual whose repertoire is in question. There are three specific points which should be made concerning this interpretation. First, what is captured by semantic matrices is taken to be a matter of formal inferences first, and logical truths verified by the matrix only second, although this is not how such matrices are usually thought of. Second, we generalize the notion of a characteristic matrix for a set of formal inferences to apply to material inferences as well. Finally, notice that in addition to the structure of material inference codified in each repertoire-matrix we can in fact identify a logic with regard to the whole idiom, insofar as some complicated conditionals will appear in all repertoires. We have not constructed a characteristic matrix for this logic by ordering the sentences of the language according to repertoire-designated conditionals. In some ways it is accordingly more appropriate to say that each repertoire expresses a single matrix valuation characteristic of a set of material inferences, and that the whole idiom comprising all admissible repertoires is characteristic of the formal or logical inferences involving the conditional we used to make explicit the materially appropriate inferences.

In this way, then, we can exploit Frege's semantic explanatory principle and the truth-oriented matrix semantics it inspired as theoretical auxiliaries useful in the formal analysis of a socially specified set of appropriate inferences. The fact that in extending the semantic tools of multivalued logic in this way to express non-logical meanings we must invoke so many multivalued (essentially one for each sentence of the language) should not be distressing. For the standard semantic interpretations of tense logic and modal logic are like this, giving each sentence a different multivalued, either a set of times or a set of possible worlds informally interpreted as the ones the sentence is true at. Seeing logic in the way I have been recommending, however, as formal tool for the explicit expression

of inferential roles, obviates the need for appealing to prior notions of truth or truth-value. We have interpreted Frege's truth-values as they figure in his semantic principle first as the designatedness values of multivalued logic, and then moving from concern with the codification of formal inference to concern with the codification of material inference, interpreted these as expressing membership in a repertoire. Recalling the social practical origins of repertoires, it would be appropriate to call the two circumstances of membership and non-membership in a particular repertoire as assertibility values with respect to that repertoire. We have given a much more precise sense to this term than semantic theorists who advocate the primacy of assertibility over truth typically manage to do, however¹⁹.

We represent the matrix valuation on the language induced by a formally expanded repertoire by associating with each sentence its repertoire-relative conceptual role, consisting of inferential circumstances and consequences of assertion. It is clear that this is an adequate representation in that this set of roles, together with the repertoire generating them, determines the partial ordering of the language by the conditional which is the Lindenbaum matrix. These conceptual roles are thus taken as multivalued, with repertoire membership identified as designatedness with respect to the semantic principle. The multivaluedness must, of course, determine compounding behavior according to our motivation. The only form of compound sentence we have so far introduced is the conditional. It is thus a criterion of adequacy of this representation that sentences with the same conceptual role, that is, multivalued, should be intersubstitutable in conditionals preserving both designatedness values and multivaluedness. Since the conceptual role of p as we have defined it clearly fixes the set of all designated conditionals in which p appears either as antecedent or as consequent and those conditionals conversely fix $/p/R$, substitution of p for q where $/p/R = /q/R$ preserves multivaluedness. Theorem 1 assures us that if p is designated, that is $p \in c(R)$, and $p \rightarrow q$ is designated, then so is q , so preserving multivaluedness preserves designatedness values. It is not the case, however, that the conceptual role of p with respect to R determines whether p is designated (assertible, taken-true) in R . We argued above that the set of designated conditionals should not determine the set of designated sentences (See also Appendix II). The roles together with the repertoire determine designatedness, of course, but since the repertoire just is the set of designated sentences, it is not remarkable that when combined with the association of sentences with roles-multivaluedness determines the designated multivaluedness. Con-

ceptual roles codify inferential relations, not the responsive dispositions associated with, for instance, the reporting use of a sentence with a particular inferential significance. We have abstracted from these considerations by limiting our attention to conceptual idioms, so fixing the inferential potentials codified in conceptual roles will not determine which sentences one becomes justificatorily responsible for save inferentially. The meanings of one's words do not determine what one believes.

"Truth-functional" connectives can now be introduced using designatedness values as the extensions of sentences. Writing the designatedness-value of p in R as $//p//_R$, so that $//p//_R=1$ if $p \in c(R)$ and is 0 otherwise, we can introduce the horseshoe by $//p \supset q//_R=1$ unless $//p//_R=1$ and $//q//_R=0$, and so on, and it will turn out that for all $R \in \mathcal{R}$ $//((p \rightarrow q) \& p) \supset q//_R=1$ (conjunction is introduced in Appendix 1) and similar tautologies will be valid. Taking repertoire-designatedness values as the primary sentential extensions, we can also introduce modal compounding in terms of the conversational accessibility relation in the standard Kripke fashion. The analogue of a necessary statement will be one which, since it is shared by all those recognized by the individual evincing the repertoire, has the status of an unjustified justifier, something which can freely be appealed to in justification, since it is accepted by one's constitutive critics. We would like to be able to semantically interpret all forms of sentential compounding by means of functions taking conceptual roles, or sets of them, into conceptual roles, as we can do for conditionals and these other simple logical connectives. Our use of the conditional as both the model of and a tool for the expression of conceptual roles embodies the belief that the contribution a sentence makes to the roles of conditionals it is a component in suffices to determine its role in other compounds.

Once again, we take our cue from Frege:

We are probably best in accord with ordinary usage if we take a judgement to be an act of judging, as a leap is an act of leaping. Of course this leaves the kernel of the difficulty uncracked, it now lies in the word 'judging'. Judging, we may say, is acknowledged to be true can only be a thought. The original kernel now seems to have cracked in two: one part of it lies in the word 'thought'

and the other in the word 'true'. Here, for sure, we must stop. The impossibility of an infinite regress in definition is something we must be prepared for in advance.²⁰

By a thought, Frege makes clear, is intended what is referred to in English by that-p clauses. We have identified these judged contents as conceptual roles. In what follows, we try to exhibit a representative variety of uses of such that-p clauses in terms of conceptual roles. We do so by first considering truth, since we are now in a better position than at the start of this essay to evaluate what is correct and what is incorrect about the view expressed above that judging or asserting is acknowledging the truth of a sentence, and since the nature of the thought involved and of the sorts of compounding it permits becomes most manifest.

Our starting-point is the prosentential theory of truth of Grover, Camp, and Belnap.²¹ That account can best be sketched as the product of three different lines of thought:

- i) the redundancy theory of Ramsey and others, which says that the conceptual content of 'it is true that-p' is always just the same as that of p. This motivation is explicit in the original article.
- ii) an account of truth in terms of infinite conjunctions and disjunctions. Although not so presented in the article, the best succinct statement of this view is in Putnam's Meaning and The Moral Sciences (pp. 15-16), considering an unpublished discussion by Stephen Leeds:

Now the following is Leeds's very neat observation:

If we had a meta-language with infinite conjunctions and infinite disjunctions (countably infinite) we wouldn't need 'true'! If we wanted to say 'what he said was true', for example, we could say: (1) (He said 'P₁' & P₁) or (He said 'P₂' & P₂) or...where the disjunction in (1) contains one disjunct for each sentence 'P_i' of the object language. But we can't, as a matter of fact, speak in infinite disjunctions. So instead we look for a finite expression equivalent to (1). Now, (2) For some x, he said x & x is true will be equivalent to (1) provided for each i (i-1,2,3...) (3) 'P_i' is true if and only P_i is correct. But this

is just Tarski's 'Criterion T'--the famous criterion of Adequacy. In short, any definition of 'true' is all right, provided (3) is satisfied.

iii) Finally, and this is what is distinctive to the view under discussion, it is observed that pronouns serve two sorts of purposes. In their lazy use, as in "Mary is still talking, as she has been for hours," they may simply be replaced by their antecedents (salva conceptual role). In their quantificational use, as in "Each positive number is such that if it is even, adding it to 1 yields an odd number," the semantic role of the pronoun is determined by a set of admissible substituends (in turn determined by the pronominal antecedent). Thus in virtue of uttering this sentence, one is committed to each sentence which results by replacing the quantificational term "Each positive number" with some admissible substituent, with admissibility determined by the anaphoric reference of the pronoun in question to its quantificational antecedent.

The prosentential theory is what you get if you decide to treat "...is true" as a syncategorematic fragment of prosentences, and then understand the new category of prosentences by analogy to other proforms, notably pronouns functioning as in (iii) above. The considerations advanced in (i) above are then seen to be correct insofar as concerns the lazy use of such prosentences. Thus "'Snow is white' is true" is read as a prosentence of laziness, having the same conceptual role as its anaphoric antecedent, in this case 'Snow is white', differing from that sentence only in the explicit acknowledgement of an anaphoric antecedent, as "She stopped" differs from "Mary stopped" when the pronoun has 'Mary' as its antecedent. The considerations advanced in (ii) above are seen as inspired by the quantificational use of prosentences formed with "...is true". Thus "Everything he said is true" is construed as a quantificational prosentence, which picks up from its anaphoric antecedent a set of admissible substituends (things that he said), and is semantically equivalent to their conjunction. The quantificational prosentence differs from such a conjunction pragmatically, again in its explicit acknowledgement of antecedents. Thus by appealing to the determination by an anaphoric antecedent of a class of admissible substituends, we can identify the conceptual role of prosentences of laziness directly with the role of its admissible substituent. For quantificational prosentences, we must appeal to the roles of "truth"-or designatedness-functional disjunctions and conjunctions to determine the role of the prosentence.

The authors of the presentential theory are concerned that "...is true" be taken to be a fragment of a presentence, not a predicate which characterizes a sentence-nominalization. This emphasis is generally well-motivated by the desire to avoid referring to propositions, but occasionally limits unnecessarily the scope of the resulting theory. In a number of places (e.g. pp. 106, 109) the theory is stated by means of an invidious contrast between the view which takes "That is true" as a whole to refer anaphorically to an antecedent sentence (the endorsed account), and one which takes reference to an antecedent to be established by nominal reference of 'that' to another sentence (the superseded view). The authors are worried that if the first part of a sentence of the form ' \emptyset is true' is taken to be a referring sentential nominalization that, first, ' \emptyset is true' will inevitably be taken to be a predicate, and second, the anaphoric presentential reference of the whole sentence will be passed over in favor of the view that the nominalization does all the referring that gets done, which would vitiate the view.

In fact this is a situation in which we can have our cake and eat it too. We consider ' \emptyset is true' as composed of a sentence nominalization \emptyset which refers to sentences, and a presentence-forming operator ' \emptyset is true'. We have specified above the conceptual role of the resulting presentence given its anaphoric antecedent. We now need to describe how the anaphoric antecedents are determined for the presentences formed with the truth operator. The general mechanism ought to be this: \emptyset refers, either independently, or itself anaphorically, to some set of sentences $R\emptyset$ (possibly a singleton). The presentence $T(\emptyset)$ (shorthand for " \emptyset is true") has a set of anaphoric antecedents (in the lazy occurrence, just a single one) which we may represent by $PS(T(\emptyset))$. The presentence-hood of $T(\emptyset)$ is then represented by the facts that:

- a) $PS(T(\emptyset))=R\emptyset$
- b) $/T(\emptyset)/_R$ is $/R\emptyset/_R$ for any R if $T(\emptyset)$ is used in the lazy way.
- c) $/T(\emptyset)/_R$ is the role of the conjunction of the $R\emptyset$, written $\&/R\emptyset/_R$ if $T(\emptyset)$ is a universal quantificational presentence,
- d) $/T(\emptyset)/_R$ is the role of the disjunction of the $R\emptyset$, written $\forall/R\emptyset/_R$ if $T(\emptyset)$ is an existential quantificational presentence.

One of the advantages of the separable presentence-forming operator is that it allows for general verb modification (hinted at by the authors). Thus suppose we have introduced some sentential operator, say negation, by defining it as a function on roles. Then its significance for presentences involving "...is true", that

is, sentences of the form $\sim T(\emptyset)$ is fixed by the equation $PS(\sim T(\emptyset)) = \{ \sim p : p \in R\emptyset \}$, together with our previous remarks. Indeed, for any sentential operator U :

$$PS(U(T(\emptyset))) = \{ U(p) : p \in R\emptyset \}$$

This principle enables us to extend any functional definition in terms of conceptual roles which we may be able to give for a sentential operator to the application of that operator to our prosentences.

The operation of "...is true" thus requires for its full specification only further account of determining $R\emptyset$ given \emptyset . Before addressing this problem, which brings us face to face with Frege's problem of 'thoughts' in the guise of that- p clauses, let us pause to remark on the significance of this general approach for Frege's inchoate account of the act of judging as acknowledging the truth of a sentence. It is an essential feature of the prosentential account of truth that we can understand the employment of the expression "...is true" only insofar as we already understand the assertional use of free-standing sentences which do not involve that expression. The roles of truth-generated prosentences are parasitic on the roles of ordinary sentences. On this account an individual will indeed "take true" all the sentences in his repertoire, that is, be justificatorily responsible for suitably constructed prosentences taking those sentences as anaphoric antecedents. But this fact cannot be used to explain what it is for some sentence to be in that repertoire. Of course for us to identify the attitude that individual has toward the beliefs codified in his repertoire we must use quantificational prosentences taking all those repertoire-designated sentences as anaphoric antecedents, and hence must use the expression "true", but this fact about the formulation of our theory does not reflect a possible order of explanation. This is a major reason that Frege's semantic assimilation of all sentences to identities involving reference to the True is defective. Our inferential-assertional approach assimilating all sentences to conditionals provides no similar temptations.

$R\emptyset$ can be determined in a number of different ways. \emptyset may be the name of a sentence, as in "'Snow is white' is true". In that case $R\emptyset$ is clearly the unit-set comprising the named sentence. Exclusive emphasis on this sort of case led to the "disquotational" theory of truth. Again, \emptyset may be a description of a sentence or set of sentences, as in "Every sentence he uttered is true", where $R\emptyset$ is determined in the obvious way, by reducing this case to the former and looking at sentences of the form "He uttered the sentence p ". More interestingly, \emptyset may itself

refer to sentences anaphorically, rather than (as in these previous cases) directly. A case like this is "If he said it, then it is true." Here anaphor comes into our story twice, once to determine $R\emptyset$, and again to determine the semantic role of $T(\emptyset)$ in terms of its prosentential anaphoric relation to $R\emptyset$. 'It' here is a pro-form, a pronoun of quantification, and hence takes its role from a class of admissible substituends determined by its antecedent, which substituends in turn determine the anaphoric antecedents of the resulting prosentence.

A case remains which cannot be so easily disposed of, however, namely that exhibited in "That Venus is an inferior planet is true" (a somewhat clumsy equivalent for "It is true that Venus is an inferior planet," which we avoid because of its distracting initial pseudo-pronoun). Cases of this sort are not those for which the prosentential theory is most helpful or most natural, since there need not in general be any antecedent to such sentences which is not contained in them (though there may be). Nor can these cases simply be assimilated to the previous ones, since no reference, either anaphoric or direct, seems to be made to sentences. We face here the problem of what Frege calls 'thoughts' in the passage quoted above. The problem is not so much with 'that'-clauses as they appear in these prosentential contexts, for we can specify their roles in these contexts immediately, but rather in the other sorts of contexts in which 'that' -clauses appear. For the class of transparent contexts, such as "That-p is true," "It is a good thing that-p," "I find it surprising that-p," and "Let us hope that it does not become the case that-p":

$$/F(\text{that-p})/_R = /F(\text{that-q})/_R \text{ iff } /p/_R = /q/_R$$

where that this hold for all p, q and $R \in \mathcal{R}$ is the criterion of the transparency of the operator F . We here specify only the contribution the that-p clause makes to the compound (which is that codified in its conceptual role).

In order to see how opaque occurrences of 'that' -clauses function, which is where difficulties arise, we shift our attention to sentential operators whose own conceptual roles are of independent interest to us. In particular, we specify the conceptual role of a sentence-forming operator $A_S(p)$, (mnemonic: S asserts p) which is informally interpreted as saying that individual S has p in his basic repertoire, that is, is prepared explicitly to undertake justificatory responsibility for p including the licensing of appropriate inferences from p . We specify another operator B_S which applies to that-clauses (mnemonic: S believes that-p), which is informally interpreted as attributing justificatory responsibility for the claim

that-p to S, whether explicitly undertaken or not, that is, saying that the claim that-p is in S's extended repertoire. These are logical operators in our special sense, in that they will enable us to express explicitly, as the contents of sentences playing particular conceptual roles, crucial structural features implicit in inferential and assertional practice, just as the conditional did. To understand such operators in terms of our formally expanded idioms and the conceptual roles they enable us to express, we must do two things. We must say what contribution the embedded sentence makes to the conceptual role of the resulting compound. For this it will suffice to specify the equivalence classes of sentences co-substitutable as components up to identity of conceptual role of the compound sentence. Second, we must say what the conceptual roles of compounds formed with these operators are, that is, describe inferential circumstances and consequences of application.

We presuppose that all of the individuals in the community using the formally expanded idiom with which we begin can recognize when another utters a particular sentence, and what sentence (syntactically individuated) it is. These recognitions are codified in the repertoire-designation of sentences of the form $U_S(q)$, where $U_S(q) \in c(R)$ if and only if (the individual characterized by) repertoire R recognizes that (the individual characterized by) repertoire S utters (or is disposed to utter) the sentence q , and we stipulate that such recognitions be correct. This assumption that community members are infallible about each other's utterances is of the same order as our assumption that individuals can recognize sentence-nominalizations such as quotations and that an anaphoric reference of one linguistic expression to another is comprehensible, or that whether one sentence occurs in another is transparent to the populace, namely broadly syntactic matters concerning the language whose use it is our project to investigate, and about which we accordingly say nothing. On the base of sentences of the form $U_S(q)$ we introduce the more interesting sentence-forming functions. We do so by imposing a set of conditions on the inferential circumstances and consequences of application of those compound sentences:

i) For all $p, q; R, S \in \mathcal{R}$, $U_S(q) \rightarrow A_S(q) \in c(R)$.

This principle specifies that if R takes S to have uttered q , then R takes S to have asserted q . In our ideal justificatory systems, assertion and inference are all that are going on, so utterances are taken to have their conventional assertional significance (though in natural languages there are various ways of modify-

ing this significance).

ii) For all $\underline{p}, \underline{q}: R \in \mathcal{R}$, $A_R(\underline{q}) \in c(R)$ iff $\underline{q} \in R$.

This principle specifies the meaning of attributions of willingness to assert to one's own case, showing that it is membership in basic repertoires that is asserted by use of the operator A_S .

iii) For all $\underline{p}, \underline{q}: R \in \mathcal{R}$, $B_R(\underline{q}) \in c(R)$ iff $\underline{q} \in c(R)$

This principle specifies for the self-regarding case that the operator B_R is to codify attributions to one's extended repertoire. This usage is extended to the case of others in accord with our previous remarks about attributions of extended responsibility by

iv) For all $\underline{p}, \underline{q}: R, S \in \mathcal{R}$, $A_S(\underline{p}) \rightarrow B_S(\underline{q}) \in c(R)$ iff $\underline{p} \rightarrow \underline{q} \in c(R)$.

This principle states that R's taking S to have asserted \underline{p} entails R's taking S to have incurred justificatory responsibility for the claim \underline{q} , just in case R is committed to the claim that \underline{p} entails \underline{q} . The motivation for the condition is that the sentence $B_S(\underline{q})$ is to codify the attribution to S of the claim that \underline{q} , as part of $c(S)$. Notice that since we have proven $\underline{p} \rightarrow \underline{p}$ valid in all formally expanded, consequence extended repertoires, it follows that $A_S(\underline{p}) \rightarrow B_S(\underline{p})$ is also valid, i.e. in every $c(R)$, for any S. This expresses the extension condition, that $R \in c(R)$. This demonstrates one direction, and motivates the other of the following biconditional:

v) For all $\underline{p}, \underline{q}: R, S \in \mathcal{R}$ $B_S(\underline{p}) \rightarrow B_S(\underline{q}) \in c(R)$ iff $\underline{p} \rightarrow \underline{q} \in c(R)$.

Our next condition specifies that the semantic condition (v) is recognized by everyone to hold (we now have the apparatus to say what this recognition consists in):

vi) For all $\underline{p}, \underline{q}: R, S, T \in \mathcal{R}$ $B_S(B_T(\underline{p}) \rightarrow B_T(\underline{q})) \in c(R)$ iff $B_S(\underline{p} \rightarrow \underline{q}) \in c(R)$.

These conditions determine the conceptual roles of statements formed using the B_R operator. It follows, for instance, from (v) and (vi) that the following formula is valid: $B_S(B_T(\underline{p} \rightarrow \underline{q})) \rightarrow B_S(B_T(B_R(\underline{p}) \rightarrow B_R(\underline{q})))$. Notice also that since $(\underline{p} \& (\underline{p} \rightarrow \underline{q})) \rightarrow \underline{q}$ is valid, so is $B_S(\underline{p} \& (\underline{p} \rightarrow \underline{q})) \rightarrow B_S(\underline{q})$ for all S, even though $\underline{p} \rightarrow \underline{q}$ may not be in some $c(R)$. (v) specifies inferential circumstances and consequences of application of $B_S(\underline{p})$ with respect to R in terms of the roles of \underline{p} and \underline{q} with respect to R, specifying that $/B_S(\underline{p})/_R = /B_S(\underline{q})/_R$ iff $/\underline{p}/_R = / \underline{q}/_R$. That is, the operator B_S thus constructed is transparent with respect to R, in virtue of being subject to these intersubstitutability conditions for its embedded component sentences.

This transparency reflects the way in which we have envisaged extended repertoires to be attributed by the relevant community. For attribution of basic repertoires, the attributor's own inferential net ought not to be decisive, so we should stipulate that:

vii) For all $\underline{p}, \underline{q}; R, S \in \mathcal{R}$, $A_S(\underline{p}) \rightarrow A_S(\underline{q}) \in c(R)$ iff $A_S(\underline{p} \rightarrow \underline{q}) \in c(R)$.

This principle states that the assertion (or assertibility) of \underline{p} by S according to R ought to be sufficient for the assertibility of \underline{q} by S according to R , just in case S , according to R , is willing to assert the conditional $\underline{p} \rightarrow \underline{q}$. The operator A_S will thus not be transparent for R , that is, intersubstitution of component sentences which have the same roles with respect to R will not in general preserve the roles of the compounds with respect to R . (vii) allows us to specify the correct inter-substitution conditions, however.

Assuming that we have identified among the logically primitive sentences we conceive the community to begin with some set of sentences to be taken as the $U_S(\underline{q})$, these principles allow us to express explicitly within the language attributions of sentences to basic and extended repertoires. To follow this out in detail would require the construction of a hierarchy parallel to that we employed in the definition of the conditional, specifying the roles of sentences involving the operators \underline{A} and \underline{B} at the first level (where they contain no embedded sentences involving those operators) according to principles (i)-(vii), and then repeating the process for the next level, taking those sentences constructed at the first level as primitive. Such a recursion on the complexity of embedded sentences will give rise to a logic of the operators investigated, as various complex sentence forms involving them and conditionals are probably valid (of the samples mentioned in the previous paragraph). These details do not concern us presently, since we want only to use this doxastic apparatus to show how the conceptual roles which the construction of the conditional enabled us to make explicit from the point of view of our theory can come to be made explicit within the idiom under investigation, thus giving rise to new expressive possibilities. In particular, we show how to develop precisely Sellars' suggestion that that- \underline{p} clauses be taken as referring to the conceptual of \underline{p} (which he expresses in by means of dot-quoted expressions).

The doxastic operators we have considered so far appeal to an ideal communi-cative use of language for the exchange of information whose interpretation is not at issue. According to principles (i) and (iv), S 's utterance of the sentence \underline{p}

will be taken by R to make S responsible for a claim which has the same significance that p has for R. Since a major feature of our account of linguistic practice is that in general the significance of p need not be the same for R as for S, we must introduce some way of expressing the recognition of such differences. What we have codified so far is the pure communicative or informational use of sentences within an idiom. Along this dimension, the exchange of sentences concerns only idioms as comprising sets of beliefs. What is at issue is the membership of certain sentences in extended repertoires, what we discussed earlier as designatedness values. We must also express the interpretive dimension of discourse, in which meanings of sentences are considered as well. The idiom must be capable of expressing as well, in the reports which constitute indirect discourse, the conceptual roles of sentences, that is, their significance with respect to different repertoires, in the sense we discussed earlier in terms of multivaluedness. As is generally acknowledged, translation is the key to the explicit codification of this interpretive dimension in indirect discourse.²²

Our construction of conceptual roles in terms of conditionals of course presents natural criteria of adequacy for translation functions between repertoires contained in a single idiom, or which are members of different idioms. Translation ought to match conceptual roles, so that, roughly, if q is taken to be the translation in repertoire S of p as used in repertoire R, then each sentence in the inferential circumstances of application of q in S should be the translation of some sentence in the circumstances of application of p according to R, and similarly for the consequences of application. Such a criterion of adequacy is necessary rather than sufficient, since for justificatory systems which are not purely inferential (that is, mathematical systems), translation involves not only the matching of inferential roles codified in formally expanded idioms, but also matching of the responsive dispositions which must be included in the circumstances of appropriate application for sentences which have a reporting use as well as an inferential one. We need concern ourselves only with inferential connections, however, and so can define role-matching between repertoires of a single idiom as follows. A function f which assigns to each sentence which is a member of a language L some sentence of that same language is a (possible) translation function between two repertoires of our idiom defined on that language. Relative to such a function, repertoires R and S are first-order role-homeomorphic at $/p/R$ and $/q/S$ just in case:

- a) $f(\underline{p})=q$
- b) For all $\underline{r} \in 1 (/p/R)$, $f(\underline{r}) \in 1 (/q/S)$
- c) For all $\underline{t} \in 1 (/q/S)$, There is a $\underline{u} \in 1 (/p/R)$ such that $f(\underline{u})=\underline{t}$.
- d) & e) Similarly for 2 ($/p/R$).

That is, the translations of the circumstances and consequences of application of \underline{p} must be the circumstances and consequences of application of the translation of \underline{p} . This is a very weak condition on a translation function, in that the roles of sentences which comprise the circumstances and consequences of application of the translation of \underline{p} may not match the roles of their translations.

The homeomorphism is so far merely local. Of course, if for every \underline{p}, q such that $f(\underline{p})=q$, R and S are first order role-isomorphic at $/p/R$ and $/q/S$, then R and S are globally isomorphic in the sense that the roles of all the sentences of the language with respect to R together with the function f determines the roles of those sentences in S, and in fact determines all of the inferential connections codified in the conditionals of $c(S)$. A more rigorous condition on conceptual roles may be defined if we shift global inferential properties into local sentential conceptual roles by constructing the following heirarchy of roles. Roles as we have already defined them will be called 'first-order' roles. The second order role of \underline{p} with respect to R will then consist of the ordered pair of the set of all first-order roles of sentences in the inferential circumstances of application of \underline{p} in R, and the set of all first-order roles of sentences in the consequences of application of \underline{p} . More formally:

$$/p/R^n = \text{df.} \quad \langle \{ /q/R^{n-1} : q \in 1 (/p/R^1) \}, \{ /r/R^{n-1} : r \in 2 (/p/R^1) \} \rangle .$$

Global homeomorphism of repertoires, representable by local homeomorphism at every \underline{p} , $f(\underline{p})$, will then be equivalent to local homeomorphism of order ω of any suitable \underline{p} , $f(\underline{p})$. Notice that the transitivity of the conditional does not make this heirarchy of roles redundant. Although the consequences of application of the consequences of application of \underline{p} are consequences of application of \underline{p} (and similarly for circumstances of application), the circumstances of application of the consequences of application of \underline{p} are neither circumstances nor consequences of application of \underline{p} and similarly for the consequences of application of the circumstances of application of \underline{p} .

Role-homeomorphisms of various degrees at various sentences constitute a crucial set of semantic facts about the structures of repertoires. If we as theorists looking at the idiom from the outside wanted to characterize communication within

the community evincing that idiom, we would employ such translation functions in determining, for instance, whether one individual understood the significance of a remark by another, since to do so includes appreciating, on our model, what the author of the remark took to follow from that remark and what he took to constitute evidence for it. The concept of local role-homeomorphism of different orders gives us a way of expressing the facts which such appreciation should reflect and embody. We show now how those semantic facts about the idiom can be expressed explicitly as the content of claims made within that idiom. We use the logical vocabulary of conditionals and repertoire attributions we have already constructed to define a further bit of expressive machinery, that-clauses, which will thus have a logical function in making explicit semantic features implicit in the idiom. Once again, as with A_R and B_R , we specify necessary conditions for identifying some previously existing locution as expressing an interpretation. In this regard the present definition differs from the construction of the conditional, which gave necessary and sufficient conditions for the logical vocabulary. As indicated earlier, it is a defining feature of logical expressions that their circumstances of appropriate application (introduction rules) together with the material practices the expressions are to codify, determine the consequences of application of those expressions. But of course the converse need not obtain--the necessary conditions need not determine the sufficient ones.

The locution we identify is represented in the form $T_S(\underline{p}:\underline{q}) \in c(R)$ and it is informally interpreted as asserting that \underline{p} in repertoire S has the same significance as \underline{q} has in R , that according to R , S 's believing \underline{p} in the sense of $\underline{p} \in c(S)$ as codified for R in $B_S(\underline{p})$ is S 's believing that- \underline{q} . T is a 'that'-mapping (or a translation-mapping). We require first that:

$$T_S(\underline{p}:\underline{q}) \in c(R) \text{ and } T_S(\underline{p}:\underline{r}) \in c(R) \text{ iff } \underline{q} \leftrightarrow \underline{r} \text{ c}(R).$$

This substitution condition guarantees that first-order conceptual roles still function as multivalued in determining roles of sentences as embedded parts of other sentences. For T to express an interpretation function (for it to be a 'that'-mapping), in an idiom it must also be the case that for any $S, R \in \mathcal{R}$, for any $\underline{p}, \underline{q}, \underline{r}, \underline{s}, \underline{t}, \underline{w}$, if $T_S(\underline{p}:\underline{q}) \in c(R)$ a condition paralleling (a) above), then first:

b') For all \underline{r} such that $B_S(\underline{r} \rightarrow \underline{p}) \in c(R)$, for all \underline{s} such that

$$T_S(\underline{r}:\underline{s}) \in c(R), \quad \underline{s} \rightarrow \underline{q} \in c(R) \quad \text{and}$$

c') For all \underline{r} such that there is an \underline{s} such that $T_S(\underline{r}:\underline{s}) \in c(R)$ and $\underline{s} \rightarrow \underline{q} \in c(R)$,

$$B_S(\underline{r} \rightarrow \underline{p}) \in c(R).$$

- d') For all \underline{t} such that $B_S(\underline{p} \rightarrow \underline{t}) \in c(R)$, for all \underline{w} such that
 $T_S(\underline{t}:\underline{w}) \in c(R)$, $\underline{q} \rightarrow \underline{w} \in c(R)$.
- e') For all \underline{w} such that there is a \underline{t} such that $T_S(\underline{t}:\underline{w}) \in c(R)$
and $\underline{q} \rightarrow \underline{w} \in c(R)$, $B_S(\underline{p} \rightarrow \underline{t}) \in c(R)$.

These conditions obviously parallel the semantic conditions on role-matching (a)-(e) above. What is matched by a locution T which is a 'that'-mapping according to these conditions is on the one hand the inferential structure R takes S to impose, as captured in statements of the form $B_S(\underline{r} \rightarrow \underline{p}) \in c(R)$ and $B_S(\underline{p} \rightarrow \underline{t})$, and on the other hand statements ordered by the conditionals in $c(R)$. Thus this is an opaque construction in its first argument place.

With respect to such a 'that'-mapping, we can then introduce richer doxastic constructions, corresponding to asserting that-p and believing that-p, as follows:

$B_S(\text{that-}\underline{q}) \in c(R)$ iff $B_S(\underline{p}) \in c(R)$ and $T_S(\underline{p}:\underline{q}) \in c(R)$, and similarly for $A_S(\text{that-}\underline{q})$.

Our general semantic strategy is to find for every sentential context $G(\underline{p})$ identified in an idiom some function which determines $/G(\underline{p})/_R$ for any $R \in \mathbb{R}$ given some interpretation function and the roles of various degrees of other sentences with respect to various repertoires. The existence of locutions meeting the conditions for 'that'-mappings determines which of the componential roles thus identified for sentences can be made explicit in the idiom in question. In an actual justificatory system, different 'that'-mappings will be introduced by different ways of matching up the responsive dispositions associated with the reporting use of sentences in various repertoires, and with various ways of extending to purely inferential sentences the partial functions such matchings generate. $A_R(\text{that-}\underline{p})$ and $B_R(\text{that-}\underline{p})$ thus codify particular forms of indirect discourse, expressing not merely the membership of sentences in various repertoires, but the place in the inferential network of another that the sentence uttered is taken by its audience to occupy. Notice that there is no assumption that individuals are omniscient about the repertoires of those whose discourse they report indirectly.

We have thus constructed not only the possibility of saying that-p, but also the possibility of saying that someone else says that-p. In my "Truth and Assertibility", such a pair of sentences as $(\underline{p}, A_S(\text{that-}\underline{p}))$ when considered from the point of view of S was called an expression-statement pair, since what an utterance of the first expresses, an utterance of the second sentence states explicitly.

In that piece the different behavior of such expressions as antecedents of conditionals was used to argue that significances had to determine both circumstances of appropriate application and the consequences of such application if the role of a sentence as free-standing utterance and as component of other sentences is to be captured. It was in terms of such expression-statement pairs that assertibility and truth were defined, and a difference funded between treating a sentence as justified (assertible) or appropriate according to its circumstances of application, on the one hand, and treating it as true on the other, in the sense of being willing to assert its inferential consequences of application. The present work has thus defined the primitive notions of assertibility and expression-statement pair which were appealed to in that earlier essay, which may accordingly be seen as an application of the current idiom and further confirmation of the utility of our notion of conceptual role.

We have developed an account of saying that-p which is unusual in starting from the practices of saying, and defining in terms of them the significant semantic contents which are what is said. We began with quite general features of social practices, and described in detail how specifically linguistic. Our theory of semantic contents is thus a pragmatic semantics, finding the ultimate source of the significance of our claims in the roles those claims play in a system of justificatory practices which we can engage in. An explicit account is given of the relation of these cognitive contents to the non-cognitive practices they arise out of. This direction of argument aside, the account of conceptual roles is novel in being entirely non-representational. In the formal idiom we develop, it is not a necessary feature of a saying that-p that the sentence involved represent some state of affairs. Of course sentences used to say things may also be representations, and this fact might be crucial for the understanding of the use of language in empirical inquiry. But our model is broader, and we may hope that it can find application in the explication of other forms of discourse (e.g. literary and political discourse) where the representationalist paradigm is less apt than it perhaps is for scientific idioms.

Perhaps the most important feature of our account is the crucial place given to logic, as providing the formal means by which an idiom can come to express explicitly crucial semantic facts which are implicit in the system of justificatory practices which are the use of a language. We argued that the function thus as-

signed to logic as a formal auxiliary in a theory of meaning is that which Frege originally envisioned and pursued. Our own development looked at the codification of inferential practice in conditionals in some detail, and somewhat less closely at the codification of repertoires in prosentences containing '...is true' and in propositional attitudes, and at the codification of roles in 'that'-clauses. The basic claim here is that logic must not be restricted to the analysis of the meanings sentences acquire in virtue of the formal inferences they are subject to (as is the usual procedure). Logic should not be viewed as an autonomous discipline in this way, but as a tool for the analysis of material inference, and for making explicit the roles played by sentences in systems of material inferential practice. Using logical devices so interpreted, we were able to specify not only what role a performance needs to play in a system of social practices in order to be a saying (asserting, professing, claiming, etc.) that-p, but also to show what it is about that system of practices in virtue of which the content of such a saying can be that someone else has said (asserted, etc.) something. Indeed the only sort of "aboutness" we ever employ is the reference of one bit of discourse to another (anaphoric reference if performance or sentence tokens are at issue, and mediated by conceptual roles otherwise).

In conclusion, we have presented a new way of talking about talking, the idiom idiom. For the sake of definiteness, specific proposals have been presented wherever possible, and it is largely in terms of these details that the viability of the project consists in the general direction of analysis recommended, the strategy employed for relating various aspects of linguistic theory to the sorts of authority and responsibility induced by social practice. An idiom can only be presented from the point of view of some repertoire, of some particular set of sentences which are asserted and justified and whose inferential potentials are specified. But the idiom containing any such repertoire is a house with many mansions, and it is the function of this essay to outline a novel idiom for talking about language. This can be accomplished even though the repertoire through which it is presented may not be the best possible for that purpose. The account of talking we have developed is thus significant primarily for the possibilities it opens up for novel firms of understanding of linguistic phenomena.

Appendix I

For all $R \in \mathcal{R}$, for all sentences \underline{p} , \underline{q} , \underline{r}

Theorem 2 - ID:

$\underline{p} \rightarrow \underline{p} \in c(R)$

Proof: This will hold at any level $k+1$ (and hence at any higher level and for I itself) just in case for every $S \in J_k(R)$ with which \underline{p} is compatible (i.e. $S+\underline{p} \in \mathcal{R}_k$), $c_k(S+\underline{p}) \subseteq c_k(S+\underline{p})$, which obviously will always hold.

Theorem 3 SPASS: $((\underline{p} \rightarrow \underline{p}) \rightarrow \underline{q}) \rightarrow \underline{q} \in c(R)$

Proof: This will hold at $k+1$ iff for every $S \in J_k(R)$ such that $(\underline{p} \rightarrow \underline{p}) \rightarrow \underline{q}$ is compatible with it, $c_k(S+\underline{q}) \subseteq c_k(S+(\underline{p} \rightarrow \underline{p}) \rightarrow \underline{q})$. According to (lii) and (3), however, $(S+(\underline{p} \rightarrow \underline{p}) \rightarrow \underline{q}) \in \mathcal{R}_k$ just in case $(\underline{p} \rightarrow \underline{p}) \rightarrow \underline{q} \in c_k(S)$. That is, though~~h~~ for atomic sentences there is a difference between \underline{p} being compatible with R and its being contained in $c_k(R)$, if \underline{p} is an arrow formula, these two are collapsed. It is this fact, stemming from our introducing the conditional as codifying pre-existing inferential relationships, which determines the main features of the logic of the arrow. Since the arrow is interpreted as expressing information about the place of a particular repertoire in a network of practices, either the circumstances for arrow introduction apply, or they do not, and if they do not, the conditional is materially incompatible with the repertoire which fails to induce it. Containment in the consequence extension is obviously sufficient for compatibility, and that it is necessary, for arrow sentences, follows from the construction of \mathcal{R}_{k+1} . So our task in this proof is to show that $c_k(S+\underline{q}) \subseteq c_k(S)$ for all $S \in J_k(R)$ such that $(\underline{p} \rightarrow \underline{p}) \rightarrow \underline{q} \in c_k(S)$. By (lii), this will hold iff for all $T \in J_{k-1}(R)$ such that $(T+\underline{p} \rightarrow \underline{p}) \in \mathcal{R}_{k-1}$, $c_{k-1}(T+\underline{q}) \subseteq c_{k-1}(T+\underline{p} \rightarrow \underline{p})$. In particular, if J_0 is reflexive, S itself satisfies the conditions for T , so we may substitute S for T , getting $c_{k-1}(S+\underline{q}) \subseteq c_{k-1}(S+\underline{p} \rightarrow \underline{p})$. By theorem 2, however, $c_{k-1}(S+\underline{p} \rightarrow \underline{p}) = c_{k-1}(S)$ (using interpolation). By the extension property and the transitivity of \subseteq , then, $\underline{q} \in c_{k-1}(S)$. Hence by (li) and interpolation $c_k(S+\underline{q}) \subseteq c_k(S)$ for any S compatible with $(\underline{p} \rightarrow \underline{p}) \rightarrow \underline{q}$, which completes the proof (by lii).

Theorem 4 SUF: $(\underline{p} \rightarrow \underline{q}) \rightarrow ((\underline{q} \rightarrow \underline{r}) \rightarrow (\underline{p} \rightarrow \underline{r})) \in c(R)$

Proof: Suppressing irrelevant subscripts, this will hold just in case $c(S+((\underline{q} \rightarrow \underline{r}) \rightarrow (\underline{p} \rightarrow \underline{r}))) \subseteq c(S+\underline{p} \rightarrow \underline{q})$ for all $S \in J(R)$ such that $\underline{p} \rightarrow \underline{q}$ is compatible with S , that is, contained in $c(S)$. For any such S , for all $T \in J(S)$ if $T+\underline{p} \in \mathcal{R}$, then $c(T+\underline{q}) \subseteq c(T+\underline{p})$. We must show that for any such S , $(\underline{q} \rightarrow \underline{r}) \rightarrow (\underline{p} \rightarrow \underline{r})$ is compat-

ible with S , that is (for arrow formulae, remember) contained in $c(S)$, which will suffice for the desired inequality stated in the first sentence of the proof. Applying the same reasoning again, $(\underline{q} \rightarrow \underline{r}) \rightarrow (\underline{p} \rightarrow \underline{r})$ will be in $c(S)$ just in case for any $T \in J(S)$, if $c(T+\underline{r}) \subseteq c(T+\underline{q})$, then $c(T+\underline{r}) \subseteq c(T+\underline{p})$. But by the above hypothesis $c(T+\underline{q}) \subseteq c(T+\underline{p})$, so by the transitivity of \subseteq , $c(T+\underline{r}) \subseteq c(T+\underline{p})$, as desired. To complete the proof, we need only show that if $T+\underline{p} \in \mathcal{R}$, then $T+\underline{q} \in \mathcal{R}$, as we presupposed several lines above. But this is guaranteed, since we know that $c(T+\underline{q}) \subseteq c(T+\underline{p})$, and hence $T+\underline{q} \in \mathcal{R}$, by hypothesis.

Theorem 5 SELFD: $(\underline{p} \rightarrow (\underline{q} \rightarrow \underline{r})) \rightarrow ((\underline{p} \rightarrow \underline{q}) \rightarrow (\underline{p} \rightarrow \underline{r})) \in c(R)$.

Proof: We have seen that the compatibility conditions for conditional statements coincide with their introduction conditions (this being a mark of their status as logical constructions), so here we must show that whenever for some $R' \in J(R)$ $\underline{p} \rightarrow (\underline{q} \rightarrow \underline{r})$ is in $c(R')$, so is $(\underline{p} \rightarrow \underline{q}) \rightarrow (\underline{p} \rightarrow \underline{r})$. Since we know that $\underline{p} \rightarrow (\underline{q} \rightarrow \underline{r}) \in c(R')$, by (lii) we know also that for all $S \in J(R')$, if $S+\underline{p} \in \mathcal{R}$, then $c(S+\underline{q} \rightarrow \underline{r}) \subseteq c(S+\underline{p})$. This requires that $(S+\underline{q} \rightarrow \underline{r}) \in \mathcal{R}$, and so we know first that $\underline{q} \rightarrow \underline{r} \in c(S)$, and second, that $c(S) \subseteq c(S+\underline{p})$, (since by interpolation and arrow compatibility conditions $c(S+\underline{q} \rightarrow \underline{r}) = c(S)$). We must use this information about all $S \in J(R')$ such that $(S+\underline{p}) \in \mathcal{R}$ to show that for all $T \in J(R')$, $c(T+\underline{p} \rightarrow \underline{r}) \subseteq c(T+\underline{p} \rightarrow \underline{q})$, whenever the right side exists (that is, when $(T+\underline{p} \rightarrow \underline{q}) \in \mathcal{R}$). So we must show that if $\underline{p} \rightarrow \underline{q} \in c(T)$, then $\underline{p} \rightarrow \underline{r} \in c(T)$. For any particular T , if $\underline{p} \rightarrow \underline{q} \in c(T)$, then we know by (lii) that for all $V \in J(T)$, if $V+\underline{p} \in \mathcal{R}$, then $c(V+\underline{q}) \subseteq c(V+\underline{p})$. We must show that for all such V , $c(V+\underline{r}) \subseteq c(V+\underline{p})$. We remark first that all such V meet the conditions put on S above, since $V+\underline{p} \in \mathcal{R}$ and $V \in J(R')$ (since $V \in J(T)$ and $T \in J(R')$ if J is transitive). So we know that $\underline{q} \rightarrow \underline{r} \in c(V)$. By (lii) this means that for all $W \in J(V)$, if $W+\underline{q} \in \mathcal{R}$, then $c(W+\underline{r}) \subseteq c(W+\underline{q})$. But now V itself meets the requirements for being a W , since $V \in J(V)$ if J is reflexive, and $V+\underline{q} \in \mathcal{R}$ is guaranteed since $c(V+\underline{q}) \subseteq c(V+\underline{p})$. Thus substituting V for W above, $c(V+\underline{r}) \subseteq c(V+\underline{q})$. And now by the transitivity of \subseteq , $c(V+\underline{r}) \subseteq c(V+\underline{p})$, which completes the proof.

Theorem 6 CNTRC: $(\underline{p} \rightarrow (\underline{p} \rightarrow \underline{q})) \rightarrow (\underline{p} \rightarrow \underline{q}) \in c(R)$

Proof: We must show that for any $S \in J(R)$, if $\underline{p} \rightarrow (\underline{p} \rightarrow \underline{q}) \in c(S)$, that $\underline{p} \rightarrow \underline{q} \in c(S)$. Since $\underline{p} \rightarrow (\underline{p} \rightarrow \underline{q}) \in c(S)$, by (lii) for any $T \in J(S)$, if $T+\underline{p} \in \mathcal{R}$, then $c(T+\underline{p} \rightarrow \underline{q}) \subseteq c(T+\underline{p})$, thus $\underline{p} \rightarrow \underline{q} \in c(T)$. Now what we must show is that for any $V \in J(S)$, if $V+\underline{p} \in \mathcal{R}$, then $c(V+\underline{q}) \subseteq c(V+\underline{p})$, (that is, that $\underline{p} \rightarrow \underline{q} \in c(S)$). But since all such V meet the T -conditions above, this is assured.

The six formulae we have just proven axiomatize EI, the pure conditional frag-

ment of Belnap and Anderson's system of entailment. The effect of EI's only rule, modus ponens, is achieved by our recursive construction, in view of IELIM, so all $c(R)$ are deductively closed under modus ponens, and hence all theorems of EI are valid in all idioms in which J meets the appropriate conditions (underlined throughout when appealed to in a proof). We now show that a prominent theorem of the pure implicational fragment S4I of S4, which is clearly undesirable in a conditional which codified pre-existing material inference, does not hold in all the logically consequence extended repertoires of all idioms (even with algebraic restrictions on J). S4I contains EI.

Theorem 7 S4IRR: It is not the case that for all $R \in \mathcal{R}$, $(\underline{p} \rightarrow \underline{q}) \rightarrow (\underline{r} \rightarrow (\underline{p} \rightarrow \underline{q})) \in c(R)$.

Proof: To construct a counter-example, we must find a conditional $\underline{p} \rightarrow \underline{q}$ which is invalid (for which there are some $R \in \mathcal{R}$ such that $\underline{p} \rightarrow \underline{q} \notin c(R)$). We will have such a counterexample if there is any $\underline{p} \rightarrow \underline{q}$ for which there are $T, V \in \mathcal{R}$ such that $\underline{p} \rightarrow \underline{q} \in c(V)$ and $\underline{p} \rightarrow \underline{q} \notin c(T)$ and $T \in J(V)$. This will suffice for our purpose since we can then let \underline{r} be any element of $c(T)$ and $c(V)$ will contain $\underline{p} \rightarrow \underline{q}$ and not $\underline{r} \rightarrow (\underline{p} \rightarrow \underline{q})$. What would be required to block the existence of such an example is clearly that for all $\underline{p} \rightarrow \underline{q}$, and for all $R, S \in \mathcal{R}$, if $\underline{p} \rightarrow \underline{q} \in c(R)$ and $S \in J(R)$, then $\underline{p} \rightarrow \underline{q} \in c(S)$. The only restriction on J which will ensure this given our construction is if for all $R \in \mathcal{R}$, $J(R) = \{R\}$. This stipulation would also entail that if $\underline{p}, \underline{q} \in c(R)$, then $\underline{p} \rightarrow \underline{q} \in c(R)$, that is, the arrow would become the familiar material conditional. It follows that for other functions J , the formula S4IRR is invalid.

Next we show that one of the basic conditions of Belnap and Anderson's \mathcal{R} does not hold in the present system. RI contains EI, and is incommensurable with S4I.

Theorem 8 ASERT: It is not the case that for all $R \in \mathcal{R}$, $\underline{p} \rightarrow ((\underline{p} \rightarrow \underline{q}) \rightarrow \underline{q}) \in c(R)$.

Proof: The validity of this formula would require that for all $S \in J(R)$, if $S + \underline{p} \in \mathcal{R}$, then $c(S + (\underline{p} \rightarrow \underline{q}) \rightarrow \underline{q}) \subseteq c(S + A)$, that is, that whenever \underline{p} is compatible with S , $(\underline{p} \rightarrow \underline{q}) \rightarrow \underline{q}$ is compatible with S , that is, is in $c(S)$. A counter-example then requires only that there be an $S \in \mathcal{R}$ and a $\underline{p}, \underline{q}$ such that $\underline{p} \rightarrow \underline{q} \in c(S)$, but $\underline{p}, \underline{q} \notin c(S)$, although $S + \underline{p} \in \mathcal{R}$. But this is generally the case, since $\underline{p}, \underline{q}$ need not be in $c(S)$ for $\underline{p} \rightarrow \underline{q}$ to be in $c(S)$.

Lest this result seem too counter-intuitive, it should be pointed out that it is a consequence of Theorem 1 that if $\underline{p} \in c(R)$, for any $R \in \mathcal{R}$, then $(\underline{p} \rightarrow \underline{q}) \rightarrow \underline{q} \in c(R)$, that is, the formula resulting from replacing the first arrow by a material conditional is a valid principle.

Because of the class differences with respect to compatibility between arrow statements and base-level statements (the difference between the formal or constructive compatibility of arrow formulae and the material compatibility of basic sentences), a restricted version of the principle of ASERT does hold, restricted namely in that p is not permitted to be a base-level statement, but must be an arrow statement. That is, for all $R \in \mathcal{R}$, $(p_1 \rightarrow p_2) \rightarrow (((p_1 \rightarrow p_2) \rightarrow q) \in c(R)$, since if $p_1 \rightarrow p_2 \in c(S)$ for some $S \in J(R)$, then $((p_1 \rightarrow p_2) \rightarrow q) \in c(S)$. In a similar way, it can be shown that permutation does not hold, that is that $(p \rightarrow (q \rightarrow r)) \rightarrow (q \rightarrow (p \rightarrow r))$ is not valid (this principle of RI entails ASERT, so no separate proof will be given). The restriction of this principle to conditional statements, however, is a theorem of EI, and is valid, namely the claim that $((p \rightarrow ((q \rightarrow r) \rightarrow s)) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow s)))$. Taking into account the differences between the treatment of base level sentences and conditionals, as antecedents and consequents of conditionals, then, it is clear that the logic determined by this semantics is the following. First, every theorem of EI is a idiomatically valid. Second, if we define a hyper-formula representation of a formula as any formula with capital sentential variables, for each of which we substitute some conditional (arrow-containing) expression, then any formula whose hyper-formula representation is a theorem of S4I (or S5I, depending on the algebraic properties of the accessibility relation J) is idiomatically valid.

Finally, we should notice that if we add truth functions to our recursive construction of the formally expanded idiom in the natural way (adding $p \& q$ to $c_{k+1}(R)$ just in case $p \in c_k(R)$ and $q \in c_k(R)$, and so on), then strenghtening of the antecedent will not be valid. That is, in general one cannot conclude that if $p \rightarrow q$, then $(p \& r) \rightarrow q$ for arbitrary r . I take denial of this principle to be essential to any context-sensitive or holistic logic. The principle is invalid in the present system since although $c(S+p) \subseteq c(S+q)$ for R-accessible S in \mathcal{R} , it may well not be the case that $c(S+p) \subseteq c(S+q+r)$, since the consequence relation is not monotonic. The material inferential connections codified formally in conditionals are defeasible. A very restricted form of the outlawed principle is valid, however. If $p \rightarrow r \in c(R)$, then if $p \rightarrow q \in c(R)$, $(p \& q) \rightarrow r \in c(R)$. This holds because if $p \rightarrow r \in c(S)$, then $r \in c(S+p)$ (if J is reflexive), and so $c(S+p+q) = c(S+p)$ by interpolation, so if $c(S+q) \subseteq c(S+p)$, then $c(S+q) \subseteq c(S+p+q)$. Of course, in this restricted form, the principle does not permit adding to an antecedent arbitrary sentences without altering the inference thereby licensed. Only sentences already inferrable from the antecedent may be added to it.

Text Continued on page (63a)

Appendix II

In this appendix we present an argument against taking the truth-functional conditional as ever adequately expressing the sense of the English "if...then___". Although in the spirit of recent such attacks such as Belnap and Anderson's Entailment (Sections 1-5), the present attack differs in that i) no other connective's behavior is relevant to our objections and ii) only sentences with a single conditional in them need be considered. Thus where objection is typically made to principles like $(p \& \neg p) \rightarrow q$ for arbitrary q , or to others like $q \supset ((p \supset p) \supset q)$, we look only at sentences of the form $p \rightarrow q$, and thus do not consider tautologies at all, objectionable or not. We show that the truth-functional conditional has the property that any consistent assignment of truth-values to all the conditionals of a language determines the truth-values of all of the propositional variables of that language. This is absurd, because no set of facts of the hypothetical form "if p then q " should determine all of the categorical facts.

Let L be a language with primitive propositional variables p, q, r , etc., and the connective \supset . Let C be the set of its first-order conditionals. We suppose that it is partitioned into two sets C_t and C_f , the first consisting of all first-order conditionals which are taken as true, and the second consisting of the rest, which are taken to be false. We designate by *C_f the set of antecedents of false conditionals, by $_*C_f$ the corresponding set of consequents, and similarly for *C_t and $_*C_t$. The question is whether such a partition of the set C of all sentences of the form $p \supset q$ determines truth-values for all the propositional variables. We use the fact that for all $p, q, r, s \in L$, if $p \supset q \in C_f$ and $p \supset r \in C_t$, then p and r must be true, and q and s must be false, since a material conditional is false just in case its antecedent is true and its consequent false. This last condition results from taking Frege's semantic principle that inference is warranted only if it does not lead from true premises to a false conclusion to express a sufficient, as well as a necessary condition for appropriate inference. We want to show that such an interpretation of the principle is a mistake. This motivates the following definition:

- A T-chain of the partition (C_f, C_t) is a sequence $c_1, c_2, \dots, c_n, \dots$ of elements of C such that
- i) $\exists c_0 \in C_f$ such that $^*c_0 = ^*c_1$ and
 - ii) $\forall n > 0 \quad ^*c_n = ^*c_{n+1}$ and $c_n \in C_t$.

From the remarks above it is clear that all the propositional variables appearing

in a T-chain are true.

An F-chain of the partition (Cf, Ct) is a sequence

$c_1, c_2 \dots c_n \dots$ of elements of C such that

i) $\exists c_0 \in Cf$ such that $*c_0 = *c_1$ and

ii) $\forall n > 0$ $*c_n = *c_{n+1}$ and $c_n \in Ct$.

The propositional variables appearing in any F-chain of the partition must be assigned the value false.

Let $U^1(Cf, Ct)$ be the set of primitive propositions (variables) appearing in some T-chain of the partition, and let $U^0(Cf, Ct)$ be the set of propositions appearing in some F-chain. Then if $U^1(Cf, Ct) \cup U^0(Cf, Ct) = PP(L)$, the set of primitive propositions of the language, each element of $PP(L)$ will get a value. Again, it is clear that this procedure will assign exactly one truth-value to each proposition just in case $U^1(Cf, Ct) \cap U^0(Cf, Ct) = \Lambda$. If the first condition obtains, we say that the partition of the conditionals spans the language L, and if it meets the second, let us say that it is consistent.

We show first that every partition (Cf, Ct) which is non-trivial (in the sense that $Cf \neq \Lambda$) spans L. For suppose not. Then there is some primitive proposition $p \in (PP(L) - (U^1(Cf, Ct) \cup U^0(Cf, Ct)))$. Let $c_0 = \underline{r} \supset \underline{s}$ be any element of Cf (which we have stipulated to be non-empty). Then the conditional $\underline{r} \supset \underline{p}$ will be an element of the set of first-order pure implications partitioned by (Cf, Ct), and hence must either be in Cf or in Ct. But it cannot be in Cf, for in that case $\underline{r} \supset \underline{p}$ is false, and in that case the definition of the conditional requires that \underline{p} be false as well. But this is contrary to our supposition that \underline{p} did not appear as antecedent or consequent in any element of an F-chain of (Cf, Ct). But \underline{p} cannot be in Ct either, for we know from the fact that $\underline{r} \supset \underline{s}$ is false that \underline{r} is true, so if $\underline{r} \supset \underline{p}$ were true, \underline{p} would be true, and would contrary to our assumption, be in some T-chain of (Cf, Ct). Thus there can be no such primitive proposition \underline{p} , and so every non-trivial partition (Cf, Ct) spans L. (I am grateful to my colleague Carl Posy, who showed me how to simplify the original version of this proof).

But not every such non-trivial partition of C is consistent, for instance the partition (C, Λ) is not. We now state necessary and sufficient conditions for a partition (Cf, Ct) to be consistent. The necessity is obvious in each case upon inspection of the definitions of T- and F-chains and the definition of consistency stated above. The conditions are:

I. $*Cf \cap *Cf = \mathcal{A}$

II. If $\underline{p} \supset \underline{q} \in Ct$ and $\underline{q} \supset \underline{r} \in Ct$, then $\underline{p} \supset \underline{r} \in Ct$.

Call a partition admissible if it satisfies these conditions. We now show that any admissible partition is consistent (and we have already shown that any non-trivial partition spans L). Again, our proof is indirect. Suppose (Cf, Ct) is an admissible partition of C which is inconsistent. By definition, then, it requires us by our fundamental principle to assign both T and F as values to some primitive proposition \underline{p} . Then there is some T-chain of (Cf, Ct) in which \underline{p} appears as an antecedent or a consequent of an element, and there is some F-chain of (Cf, Ct) in which \underline{p} appears as the component of an element. That is, there is some \underline{p} such that:

a) There is some $c_0 = \underline{r} \supset \underline{s} \in Cf$ such that there are c_1, \dots, c_k all elements of Ct such that $*c_1 = \underline{r}$ and $\forall n > 0 \quad *c_n = *c_{n+1}$ and $*c_k = \underline{p}$.

b) There is some $c_0' = \underline{r}' \supset \underline{s}'$ such that there are $c_1', \dots, c_m' \in Ct$ such that $*c_1' = \underline{s}'$ and $\forall n > 0 \quad *c_n' = *c_{n+1}'$ and $*c_m' = \underline{p}$. But now consider the conjoined sequence of conditionals $c_0, c_1, \dots, c_k, c_m', c_{m-1}', \dots, c_1', c_0'$. All of these conditionals will be elements of Ct (except c_0, c_0') by (a) and (b) above. Further by construction that sequence has the following form:

$$\underline{r} \supset \underline{s}, \underline{r} \supset \underline{t}_1, \underline{t}_1 \supset \underline{t}_2, \dots, \underline{t}_{k-1} \supset \underline{p}, \underline{p} \supset \underline{w}_{m-1}, \underline{w}_{m-1} \supset \underline{w}_{m-2}, \dots, \underline{w}_1 \supset \underline{r}', \underline{r}' \supset \underline{s}'.$$

Now by principle (II) above, this entails that $\underline{r} \supset \underline{s}' \in Ct$. But since both $\underline{r} \supset \underline{s}$ and $\underline{r}' \supset \underline{s}'$ are in Cf, \underline{r} must be true and \underline{s}' must be false, and so $\underline{r} \supset \underline{s}' \in Cf$ as well, by the definition of the conditional. But since (Cf, Ct) is a partition of C, $\underline{r} \supset \underline{s}'$ cannot be in both Cf and Ct, so there can be no such \underline{p} as we hypothesized. So given (I), we conclude that $U^1(Cf, Ct) \cap U^0(Cf, Ct) = \mathcal{A}$.

As indicated in the text, the connection between holism and the denial of antecedent-strengthening is this. For material inference, the claim is that if q can be inferred from any finite set of atomic premises $p_1 \dots p_n$, written $p_1, \dots, p_n \Vdash q$, then it will always be possible to find some atomic r such that $p_1, \dots, p_n, r \not\vdash q$. Any particular material inference has an indefinite number of defeasibility conditions which are denials of (or at least materially incompatible with) crucial elements of the ceteris paribus clauses implicit in any finitely statable inference. "If I miss this plane, then I will arrive late for the meeting in Boston," is an appropriate inference only if the military does not suddenly put supersonic transport at my disposal, and time doesn't vary with location in heretofore unheard of ways, and so on. Put another way, every material inference from a finite number of premises has an indefinite number of semantic presuppositions, the denial of any one of which would impair the inference. As Wittgenstein argues in the Investigations, we cannot in principle make all such presuppositions explicit (there is no definite totality of possibly inference-defeating auxiliary hypotheses whose denials could be added to our premises). This fact makes material inference holistic in that the statement of any inference $p_1 \dots p_n \Vdash q$ must implicitly refer to a context of available auxiliary hypotheses, and is appropriate only insofar as that context contains no defeasors. Material inference is thus relative to a set of sentences, and depends upon that set's not including defeasing r . This is a total evidence condition. Holism in the sense of "Two Dogmas of Empiricism" is the fact that inferences involve such relativity to contexts (and the claims those contexts exclude), and hence entails the denial of antecedent strengthening. One may, with Quine and most of the tradition, conclude that we can only formalize the atomistic inferences constitutive of our own logical vocabulary, and eschew the representation of holistic material inference and hence of non-logical meanings. Our construction shows at least that this is not the only response available to the recognition of the context-sensitivity of inferential justification outside of a purely logical setting.

Notes to Text

- 1.) This is Frege's strategy, and that of most of those who employ possible-world constructions in formalizing semantic theory. For an overview, see Harman's "Three Levels of Meaning" J. Phil 65, pp 590-601 (D.'68).
- 2.) These remarks apply equally to the attempt to derive accounts of pragmatic -ings (such as calling true, predicating, referring, informing) from semantic -eds, however represented.
- 3.) Grice: "Meaning" Phil. Rev. vol 66, pp. 377-88.
Lewis: Convention Harvard U. Press, Cambridge 1969, and "Languages and Language" in Gunderson (ed.) Minnesota Studies in the Philosophy of Science, vol VII 1976.
Searle: Speech Acts Cambridge U. Press 1969.
- 4.) Harman: Thought Princeton U. Press 1973.
Putnam: "Realism and Reason" and "Reference and Understanding" in Putnam's Meaning and the Moral Sciences (Routledge, Kegan, Paul, London 1978) and "Language and Reality" in Mind, Language, and Reality--Philosophical Papers Vol. 2 Cambridge U. Press 1975.
Bennett: Linguistic Behavior Cambridge U. Press 1977.
- 5.) Defining practices in this way is motivated by:
 - a) Sellars' identification (e.g. in "Some Reflections on Language Games" in Science, Perception, and Reality, Routledge, Kegan, Paul, London 1963) of rules of criticism as prior to rules codifying individual ought-to-do's which are ordinarily but incorrectly thought of as the "rules of language"; and
 - b) Wittgenstein's demonstration in the Investigations that a background of uncodified public practices is presupposed by the application of any rule.
- 6.) I have discussed the consequences of this observation for the ontological status of practices in "Freedom and Constraint by Norms" American Philosophical Quarterly July 1979.
- 6.5) In the crucial section on Lordship and Bondage of the Phenomenology, Hegel defines the passage from isolated consciousness to self-consciousness correlative to a community as occurring when two individuals "...recognize themselves as mutually recognizing one another." (Baille translation, p. 231). In the section on Conscience, he says:

The totality or actuality which is revealed as the truth of the ethical world, the world of the social order, is the self of the Person: its ex-

istence lies in being recognized and acknowledged." (B; p. 645)

Introducing a theme developed in this essay, he says:

Language, however, comes forward merely as the mediating element only between self-consciousnesses independent and recognized; and the existent self means immediately universal recognition, means manifold recognition and in this very manifoldness simple recognition." (B; p. 611).

- 7.) Lewis analyzes this feature of assertion in terms of conventions of truthfulness and trust in a community.
- 8.) Searle, Speech Acts, p. 66
- 9.) In particular, it is not clear how to build an unrelativized notion of being responsible for on this basis. It is not our project to account for all the uses of the expressions 'responsibility' and 'authority'.
- 10.) "Empiricism and the Philosophy of Mind" (in Science, Perception and Reality, see note 5), section 38. The view propounded here is an exploration of the "dimension of endorsement" which Sellars in that essay shows to be of crucial importance for epistemology and the philosophy of language (see, e.g., section 16).
- 11.) Since the individuals who cognitively generate the conversational accessibility relation are represented in an idiom only by their repertoires, we require that all differences of accessibility correspond to differences in repertoire.
- 11.5) It is important that the linguistic system we attribute to an individual should represent necessary, not sufficient conditions on utterances. We don't want to explain away the fact that almost every sentence uttered by an adult native speaker of a natural language is the first tokening ever of that sentence-type. On our model, only sentences in one's repertoire are assertible, but this only constrains and does not determine actual utterances.
- 11.75) The idea here is Sellars', that material inferential practices underwrite content-rules for a language (see for instance "Some Reflections on Language Games"). Perhaps the best extended discussion of this theme is Chapter 3 of Rosenberg's Linguistic Representation (D. Reidel, Dordrecht, 1975).
- 12.) We assume only that the inferences one may draw from p , together with $\{q\} \cup S$ are the same as those one may draw from q , together with $\{p\} \cup S$.
- 13.) Strictly speaking, sentences will not appear as antecedents if they are materially incompatible with all accessible repertoires (that is, $S+p \not\subseteq R$ for all $S \in J(R)$). They will not appear as consequents if they don't appear in any extended repertoires accessible from R , or from admissible supersets of R .

13.5) Notice that it is a consequence of our construction that the set of consequents of conditionals in any R which have the same antecedent ($\{ q: p \rightarrow q \in c(R) \}$) will be a compossible set, in that there will be some R' whose consequence extension contains that set. This is clearly a desirable property of counterfactual conditionals. Herzberger argues ("Counterfactuals and Consistency", J. Phil. 76, number 2- F. 1979) that Lewis' counterfactuals (Counterfactuals, Harvard U. Press 1973) do not meet this condition.

13.75) Our notion of a conceptual idiom thus gives content to the nineteenth century notion of cognitive perspectives (see Nietzsche's remarks about "perspectivism" in the Will to Power, for instance). Nietzsche's idea was that as visual objects could be thought of as consisting in all of the possible visual perspectives on them, so the objects of cognition could be thought of as consisting of all possible cognitive perspectives on them. The metaphor could not be cashed, however, for no independent content had been given to the notion of a "cognitive" perspective. Idiomatic propositions in our scheme (functions from repertoires to repertorial roles), however, are just such perspectival 'objects', with each repertoire a possible cognitive "point of view", and the idiom itself as the system of similarities and differences in what is assertible from each point of view which gives meaning to the notion of "perspective". This theme will be developed at greater length in the account of sub-sentential reference I hope to present in a companion piece to the present one.

14.) See also "Abstract Entities" in Philosophical Perspectives published by Charles C. Thomas, Springfield Illinois, 1967, and "Some Reflections on Language Games"

15.) Frege: Philosophy of Language Harper & Row, New York 1973, e.g., at page 397. The model of meaning as consisting of the two parts of circumstances and consequences of application which is employed in this paper is due to Dummett:

Crudely expressed, there are always two aspects of the use of a given form of sentence: the conditions under which an utterance of that sentence is appropriate, which include, in the case of an assertoric sentence, what counts as an acceptable ground for asserting it; and the consequences of an utterance of it, which comprise both what the speaker commits himself to by the utterance and the appropriate response on the part of the hearer, including, in the case of assertion, what he is entitled to infer from it if he accepts it. (p. 396)

The present essay results from a synthesis of this and related ideas of Dummett's with the Sellarsian account of semantics and epistemology.

Typographical Errata in

"Assertion and Conceptual Roles"

- p. 5, 1.25: "norms of conduct"
- p. 10, 1.16: "Suppose that there is some kind of performance K' such that there is a set P of possible performances..."
- p. 17, 1.5: "The authority of reasons (our justificatory assertions)"
1.19: "...set justification off from other responsibility-discharging performances such as fulfilling promises)."
- p. 20, 1.5: "...attribution of an idiom to a community..."
- p. 21, 1.2 from bottom: "...which the assertion of one sentence (which becomes the antecedent of the conditional)..."
- p. 25, 1.11: "...where we have written "S+q" for Sq"
- p. 29, 1.6: "...if p is in some consequence-extended repertoire, and p→q is also in that repertoire..."
- p. 41, 1.8: "...the two circumstances of membership and non-membership in a particular repertoire assertibility values..."
- p. 42, ~~ix~~ bottom--Frege citation should conclude: "Judging, we may say, is acknowledging the truth of something; what is acknowledged to be true can only be a thought. The original kernel now seems to have cracked in two; one part of it lies in the word 'thought' and the other in the word 'true'..."
- p. 50, 1.24: "...~~x~~ involving them and conditionals are provably valid..." (not 'probably')
- 1.31: "...that-p clauses be taken as referring to the conceptual role of p..."
- p. 53, 1.26: "... iff q←x ∈ c(R)."
- p. 55, 1.17: "...described in detail how specifically linguistic practices develop out of them."
- p. 56, 1.22: "For the sake of definiteness, specific proposals have been presented wherever possible, and it is largely in terms of these details that the viability of the ~~project~~ overall project must be evaluated. But the importance of the ~~project~~ project consists in the general direction of analysis recommended, the strategy employed for relating various aspects of linguistic ~~theory~~ theory to the sorts of authority and responsibility ~~induced~~ induced by social practice."